

Error Pattern Exercises Solutions

Determine the error pattern the student made in each of the following problems:

$$46.325 + 234.56 + 13.567 + 2.7964 = 111.312$$

$$3.579 + 54.32 + 684.2 = 158.53$$

$$35.234 + 67.531 = 102.765$$

$$4.8 + 32 + 0.79 + 7.8 = 23.7$$

Describe the error the student is making. List the steps you would employ to assist the student in learning how to do the problem correctly and avoid repeating the same error. Could this error have been caused because the students are not accustomed to seeing addition problems written horizontally?

Solution: The addends are aligned by the right digit. Addition is correct. The average number of decimal places from the addends are transferred to the sum. Teaching students to line up the ones digit (not the decimal point, which is an unnecessary rule), solves the dilemma.

Determine the error pattern the student made in each of the following problems:

$$46.325 + 234.56 + 13.567 + 2.7964 = 0.000000111312$$

$$3.579 + 54.32 + 684.2 = 0.015853$$

$$35.234 + 67.531 = 0.102765$$

$$4.8 + 32 + 0.79 + 7.8 = 0.0237$$

Describe the error the student is making. List the steps you would employ to assist the student in learning how to do the problem correctly and avoid repeating the same error.

Solution: The addends are aligned by the right digit. Addition is correct. The total number of decimal places from the addends are transferred to the sum, necessitating the addition of so many zeros in the first sum. Teaching students to line up the ones digit (not the decimal point, which is an unnecessary rule), solves the dilemma.

Determine the error pattern the student made in each of the following problems:

$$\begin{array}{r} 4567 \\ + 7968 \\ \hline 14635 \end{array}$$

$$\begin{array}{r} 389 \\ + 964 \\ \hline 1453 \end{array}$$

$$\begin{array}{r} 2468 \\ + 3517 \\ \hline 7085 \end{array}$$

$$\begin{array}{r} 3421 \\ + 2476 \\ \hline 5897 \end{array}$$

Solution: Regrouping is carried across to each place to the left, accumulating as it goes. For example, in $4567 + 7968$, the regrouped "1" (1 ten) is shown in the tens, hundreds, and thousands columns. Likewise, the "1" (1 hundred) is shown in the hundreds, and thousands columns. Finally, the "1" (1 thousand) regrouped from the hundreds is shown in the thousands column. So, in the thousands column, when the 7 and 4 are added, THREE regrouped 1s are added to the sum instead of the "normal" one. Place value understanding would help correct this difficulty quickly.

Each of the following set of problems involving whole numbers is done incorrectly the same way. Determine the error pattern, solve the 4th and 5th problems using that pattern, describe it in your own words, indicate possible reasons for the

Error Pattern Exercises Solutions

error, and describe ways to help the student overcome the misconception (for more information on error patterns, see Ashlock, 1998).

a.
$$\begin{array}{r} 23 \\ + 4 \\ \hline 9 \end{array}$$

$$\begin{array}{r} 64 \\ + 5 \\ \hline 15 \end{array}$$

$$\begin{array}{r} 27 \\ + 4 \\ \hline 13 \end{array}$$

$$\begin{array}{r} 82 \\ + 9 \\ \hline \end{array}$$

$$\begin{array}{r} 34 \\ + 6 \\ \hline \end{array}$$

Solution: 19, 13. The student is adding digits and disregarding place value. Among other reasons, it is possible that student is confusing this exercise with column addition. Remediation in the areas of place value and regrouping at the concrete level is needed.

b.
$$\begin{array}{r} 38 \\ + 47 \\ \hline 715 \end{array}$$

$$\begin{array}{r} 52 \\ + 83 \\ \hline 135 \end{array}$$

$$\begin{array}{r} 27 \\ + 39 \\ \hline 516 \end{array}$$

$$\begin{array}{r} 64 \\ + 59 \\ \hline \end{array}$$

$$\begin{array}{r} 81 \\ + 28 \\ \hline \end{array}$$

Solution: 1113, 109. The student is disregarding regrouping. The second problem is correct because no regrouping is involved. Among other reasons, student might have some misunderstanding of the concept of place value. The use of base 10 blocks could help.

c.
$$\begin{array}{r} 342 \\ + 631 \\ \hline 973 \end{array}$$

$$\begin{array}{r} 1 \\ 74 \\ + 43 \\ \hline 18 \end{array}$$

$$\begin{array}{r} 4 \\ 385 \\ + 667 \\ \hline \end{array}$$

$$\begin{array}{r} 282 \\ + 723 \\ \hline \end{array}$$

$$\begin{array}{r} 279 \\ + 836 \\ \hline \end{array}$$

Solution: 915, 1516. The student is adding from left to right and regrouping the incorrect digit. This might be because this is the way we read. The first exercise is correct because no regrouping is involved. Estimating the answer might help in developing better number sense.

d.
$$\begin{array}{r} 32 \\ - 16 \\ \hline 16 \end{array}$$

$$\begin{array}{r} 245 \\ - 137 \\ \hline 112 \end{array}$$

$$\begin{array}{r} 524 \\ - 298 \\ \hline 374 \end{array}$$

$$\begin{array}{r} 458 \\ - 372 \\ \hline \end{array}$$

$$\begin{array}{r} 241 \\ - 96 \\ \hline \end{array}$$

Solution: 126, 245. Always subtracting the smaller number from the larger number. The first problem involving two digit numbers was correct. Some mental computation comparing the two numbers might have been done. Either correcting the steps to following for a left to right algorithm needs to be developed properly or the development of another correct type of subtraction algorithm should be provided. The borrow pay back or the decomposition methods could be alternatives.

e.
$$\begin{array}{r} 578 \\ + 179 \\ \hline 647 \end{array}$$

$$\begin{array}{r} 479 \\ + 578 \\ \hline 947 \end{array}$$

$$\begin{array}{r} 554 \\ + 256 \\ \hline 700 \end{array}$$

$$\begin{array}{r} 195 \\ + 589 \\ \hline \end{array}$$

$$\begin{array}{r} 355 \\ + 256 \\ \hline \end{array}$$

Error Pattern Exercises Solutions

Solution: 674, 501. The student is disregarding the regrouping from ones to tens. The student might need help with understanding place value and how it is used in the regrouping process.

f.

$$\begin{array}{r}
 8\ 17 \\
 \cancel{19}\ \cancel{7} \\
 -\ 4\ 3 \\
 \hline
 1414
 \end{array}
 \qquad
 \begin{array}{r}
 6\ 16 \\
 \cancel{17}\ \cancel{6} \\
 -\ 2\ 3 \\
 \hline
 1413
 \end{array}
 \qquad
 \begin{array}{r}
 7\ 14 \\
 \cancel{38}\ \cancel{4} \\
 -\ 5\ 9 \\
 \hline
 325
 \end{array}
 \qquad
 \begin{array}{r}
 273 \\
 -\ 51 \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 385 \\
 -\ 39 \\
 \hline
 \end{array}$$

Solution: The student is regrouping when it is not needed. The third problem is correct because regrouping was necessary. The student might have overgeneralized the regrouping process.

g.

$$\begin{array}{r}
 8 \\
 \cancel{19}1 \\
 -\ 43 \\
 \hline
 443
 \end{array}
 \qquad
 \begin{array}{r}
 2 \\
 \cancel{3}25 \\
 -\ 151 \\
 \hline
 174
 \end{array}
 \qquad
 \begin{array}{r}
 5\ 12\ 16 \\
 \cancel{7}\ \cancel{2}\ \cancel{6} \\
 -\ 3\ 4\ 9 \\
 \hline
 2\ 8\ 7
 \end{array}
 \qquad
 \begin{array}{r}
 638 \\
 -\ 349 \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 638 \\
 -\ 129 \\
 \hline
 \end{array}$$

Solution: 499, 509. The student has no difficulties solving exercises involving regrouping only one regrouping (from tens to ones or from hundreds to tens). The student does not perform correctly exercises involving regrouping from tens to ones and hundreds to tens at the same time. In this case, the student will take two hundreds and incorrectly distribute them as tens and ones.

h.

$$\begin{array}{r}
 2 \\
 23 \\
 +\ 39 \\
 \hline
 71
 \end{array}
 \qquad
 \begin{array}{r}
 53 \\
 +\ 26 \\
 \hline
 79
 \end{array}
 \qquad
 \begin{array}{r}
 3 \\
 28 \\
 +\ 45 \\
 \hline
 91
 \end{array}
 \qquad
 \begin{array}{r}
 45 \\
 +\ 35 \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 48 \\
 +\ 36 \\
 \hline
 \end{array}$$

Solution: 71, 111. The student is reversing the proper order of the place value for regrouping purposes. The student might have mismemorized the steps of the algorithm. Understanding of place value is a major concern.

$$\begin{array}{r}
 313 \\
 \times\ 4 \\
 \hline
 1252
 \end{array}
 \qquad
 \begin{array}{r}
 210 \\
 \times\ 15 \\
 \hline
 210
 \end{array}
 \qquad
 \begin{array}{r}
 524 \\
 \times\ 34 \\
 \hline
 1576
 \end{array}
 \qquad
 \begin{array}{r}
 135 \\
 \times\ 463 \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 345 \\
 \times\ 36 \\
 \hline
 \end{array}$$

Solution: 485, 920. The student is disregarding the regrouping process and partial products. The might be thinking about the addition computation algorithm and made the improper generalizations.

j.

$$\begin{array}{r}
 32 \\
 \times\ 3 \\
 \hline
 6 \\
 \underline{\quad} \\
 9
 \end{array}
 \qquad
 \begin{array}{r}
 42 \\
 \times\ 4 \\
 \hline
 8 \\
 \underline{\quad} \\
 16
 \end{array}
 \qquad
 \begin{array}{r}
 31 \\
 \times\ 8 \\
 \hline
 8 \\
 \underline{\quad} \\
 24
 \end{array}
 \qquad
 \begin{array}{r}
 23 \\
 \times\ 3 \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 42 \\
 \times\ 3 \\
 \hline
 \end{array}$$

Error Pattern Exercises Solutions

$$15 \qquad 24 \qquad 32$$

Solution: 18, 24. The student is treating the numbers as single digits and disregarding place value and partial products.

$$\text{k. } \begin{array}{r} 34 \\ \times 2 \\ \hline 68 \end{array} \quad \begin{array}{r} 27 \\ \times 4 \\ \hline 88 \end{array} \quad \begin{array}{r} 18 \\ \times 3 \\ \hline 34 \end{array} \quad \begin{array}{r} 24 \\ \times 4 \\ \hline \end{array} \quad \begin{array}{r} 35 \\ \times 3 \\ \hline \end{array}$$

Solution: 86, 95. The student might be forgetting to add the regrouped tens to the second partial product. The exercises not requiring regrouping are correct. Demonstrations using base 10 blocks and repeated addition model could be used here.

$$\text{l. } \begin{array}{r} 27 \\ \times 5 \\ \hline 255 \end{array} \quad \begin{array}{r} 43 \\ \times 6 \\ \hline 308 \end{array} \quad \begin{array}{r} 62 \\ \times 7 \\ \hline 494 \end{array} \quad \begin{array}{r} 38 \\ \times 6 \\ \hline \end{array} \quad \begin{array}{r} 28 \\ \times 5 \\ \hline \end{array}$$

Solution: 428, 300. The student is incorrectly adding the regrouped tens and multiplying this sum. The student might have misremembered the step required in the algorithm. Demonstrations using base 10 blocks and repeated addition model could be used here.

$$\text{m. } \begin{array}{r} 233 \\ 2 \overline{)176} \end{array} \quad \begin{array}{r} 211 \\ 4 \overline{)824} \end{array} \quad \begin{array}{r} 231 \\ 3 \overline{)713} \end{array} \quad \begin{array}{r} \\ 3 \overline{)639} \end{array} \quad \begin{array}{r} \\ 4 \overline{)518} \end{array}$$

Solution: 213, 142. The student is always dividing the smaller number into the larger number and disregarding the remainders. The second and fourth problems are correct.

$$\text{n. } \begin{array}{r} 33 \\ 3 \overline{)99} \\ \hline 90 \\ \hline 9 \\ \hline 9 \\ \hline 0 \end{array} \quad \begin{array}{r} 25 \\ 7 \overline{)364} \\ \hline 35 \\ \hline 14 \\ \hline 14 \\ \hline 0 \end{array} \quad \begin{array}{r} 78 \\ 8 \overline{)696} \\ \hline 64 \\ \hline 56 \\ \hline 56 \\ \hline 0 \end{array} \quad \begin{array}{r} \\ 4 \overline{)192} \end{array} \quad \begin{array}{r} \\ 6 \overline{)528} \end{array}$$

Solution: 84 (instead of 48), 88 (apparently correct). Improperly placing the digits of the final answer by reversing the ten and one places. This is an improper implementation of the algorithm. The student might have misremembered the steps of the algorithm.

Each of the following set of problems involving fractions is done incorrectly the same way. Determine the error pattern, solve the 4th and 5th problems using that pattern, describe it in your own words, indicate possible reasons for the error, and describe ways to help the student overcome the misconception (for more information on error patterns, see Ashlock, 1998).

Error Pattern Exercises Solutions

a. $\frac{4}{5} + \frac{1}{2} = \frac{5}{7}$ $\frac{1}{4} + \frac{1}{4} = \frac{2}{8}$ $\frac{1}{6} + \frac{3}{4} = \frac{4}{10}$ $\frac{3}{4} + \frac{1}{3} =$ $\frac{5}{8} + \frac{1}{9} =$

Solution: $\frac{4}{7}, \frac{6}{17}$. The student is not finding the common denominators and equivalent fractions before adding them.

b.

$$\begin{array}{r} \frac{1}{2} = \frac{2}{4} \\ + \frac{1}{4} = \frac{2}{4} \\ \hline \frac{4}{4} = 1 \end{array} \quad \begin{array}{r} \frac{1}{3} = \frac{3}{9} \\ + \frac{2}{3} = \frac{6}{9} \\ \hline \frac{9}{9} = 1 \end{array} \quad \begin{array}{r} \frac{3}{5} = \frac{6}{10} \\ + \frac{1}{10} = \frac{2}{10} \\ \hline \frac{8}{10} = \frac{2}{5} \end{array} \quad \begin{array}{r} \frac{1}{7} = \\ + \frac{3}{14} = \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{5} = \\ + \frac{5}{6} = \\ \hline \end{array}$$

Solution: $\frac{8}{14} = \frac{4}{7}, \frac{18}{3} = 6$ The student is converting fractions improperly and when it is not necessary. The student might need to work with equivalent fractions at the concrete level.

c.

$$\begin{array}{r} 3\frac{1}{2} = \frac{3}{6} \\ + 2\frac{1}{3} = \frac{2}{6} \\ \hline \frac{5}{6} \end{array} \quad \begin{array}{r} 2\frac{2}{5} = \frac{4}{10} \\ + 8\frac{2}{10} = \frac{2}{10} \\ \hline \frac{6}{10} \end{array} \quad \begin{array}{r} 8\frac{1}{4} = \frac{5}{20} \\ + 7\frac{3}{5} = \frac{12}{20} \\ \hline \frac{17}{20} \end{array} \quad \begin{array}{r} 5\frac{1}{6} \\ + 3\frac{3}{12} \\ \hline \end{array} \quad \begin{array}{r} 3\frac{1}{5} \\ + 2\frac{3}{6} \\ \hline \end{array}$$

Solution: $\frac{5}{12}, \frac{21}{30} = \frac{7}{10}$. The student is disregarding the whole number part of the mixed number and adding the fractions only. The student needs to work with converting mixed numbers into improper fractions before adding the fractions.

d.

$$\frac{1}{8} \times 1 = \frac{1}{8} \quad \frac{2}{3} \times 3 = \frac{6}{9} \quad \frac{4}{5} \times 2 = \frac{8}{10} \quad \frac{3}{9} \times 4 = \quad \frac{1}{9} \times 5 =$$

Solution: $\frac{12}{36}, \frac{5}{45}$. The student is multiplying the fraction numerator and denominator by the whole number (1, 3, 2, 4 or 5, respectively) and will essentially get an equivalent fraction. The student is multiplying the fraction by one instead of using the fraction form of the whole number (which is the whole number over one). The first exercise is correct because we happen to be multiplying by one. The student needs to work on converting whole numbers to a fraction and what this implies.

Error Pattern Exercises Solutions

e.

$$\frac{2}{3} \times \frac{3}{5} = 90 \quad \frac{1}{5} \times \frac{3}{4} = 60 \quad \frac{2}{3} \times \frac{2}{5} = 60 \quad \frac{3}{6} \times \frac{1}{7} = \quad \frac{5}{6} \times \frac{3}{7} =$$

Solution: 126, 630. The student is multiplying all the fraction numerators and denominators as single factors to get the final product. The student needs to work with the meaning of multiplying fractions, and the difference between multiplying whole numbers and proper fractions.

Each of the following set of problems involving decimals is done incorrectly the same way. Determine the error pattern, solve the 4th and 5th problems using that pattern, describe it in your own words, indicate possible reasons for the error, and describe ways to help the student overcome the misconception (for more information on error patterns, see Ashlock, 1998).

a.

0.3 <u>+0.9</u> 0.12	0.4 <u>+0.7</u> 0.11	0.4 <u>+0.8</u> 0.32	0.5 <u>+0.6</u>	0.9 <u>+0.9</u>
----------------------------	----------------------------	----------------------------	--------------------	--------------------

Solution: 11, 18. The student might be using the incorrect algorithm when placing the decimal point for the sum. The student might be counting the number of decimal places as you do in multiplication of decimals.

b.

3.69 <u>- 2.8</u> 1.1	5.32 <u>- 4.3</u> 1.9	7.18 <u>- 3.5</u> 4.3	8.97 <u>- 5.8</u>	6.34 <u>- 4.3</u>
-----------------------------	-----------------------------	-----------------------------	----------------------	----------------------

Solution: 3.9, 2.1. The student is disregarding the tenths place in the decimal number and subtracting across the tenths from the hundredths. When needed, the student seems to be regrouping from the tenths to the hundredths before subtracting.

c.

4 12 5.32 <u>- 0.08</u> 4.34	6 12 7.22 <u>- 0.06</u> 6.26	4 14 5.34 <u>- 0.09</u> 4.35	7.67 <u>- 0.08</u>	9.85 <u>- 0.08</u>
--	--	--	-----------------------	-----------------------

Solution: 6.89, 8.87. The student is regrouping from the ones to the hundredths, skipping the tenth place value altogether. The student has probably mismemorized the steps for the algorithm.

d.

2.7 <u>x 0.6</u> 16.2	8.36 <u>x 6</u> 50.16	0.765 <u>x 2.6</u> 4590 <u>1530</u>	4.64 <u>x 0.5</u>	5.65 <u>x 7</u>
-----------------------------	-----------------------------	--	----------------------	--------------------

Error Pattern Exercises Solutions

19.890

Solution: 23.20, 39.55. The student is making an incorrect placement of the decimal point for the final product. The student is probably bringing down the decimal point as you do with addition of decimals. In some cases, the answer is correct.

e.
$$\begin{array}{r} 0.543 \\ 6 \overline{) 3.27} \\ \underline{30} \\ 27 \\ \underline{24} \\ 3 \end{array}$$

$$\begin{array}{r} 9.062 \\ 4 \overline{) 36.26} \\ \underline{36} \\ 26 \\ \underline{24} \\ 2 \end{array}$$

$$\begin{array}{r} 27.871 \\ 3 \overline{) 83.62} \\ \underline{6} \\ 23 \\ \underline{21} \\ 26 \\ \underline{24} \\ 22 \\ \underline{21} \\ 1 \end{array}$$

$$\begin{array}{r} \overline{) 78.65} \\ 4 \end{array}$$

$$\begin{array}{r} \overline{) 78.68} \\ 5 \end{array}$$

Solution: 19.661, 15.733. The student is incorrectly placing the remainder as part of the final answer (quotient).

Reference

Ashlock, R. (2005). *Error Patterns in Computation*. Upper Saddle River, NJ: Prentice Hall.

Brumbaugh, D. K., Ortiz, E., Gresham, G. (2006). *Teaching Middle School Mathematics*. Mahwah, NJ: Lawrence Erlbaum Associates.

Brumbaugh, D., Rock, D. (2001). *Scratch Your Brain C1*. Pacific Grove, CA: Critical Thinking Books and Software.