Select a topic, like “What percent of 8 is 5?” and research how to teach it and all the prerequisite experiences the student should have prior to broaching this subject. Describe the skills you would expect the student to have prior to the study of the topic and the method you would use to introduce it.

Solution: Answers will vary

Where do you stand on the use of a calculator, and why?

Solution: Answers will vary

Present your opinion of the Mathematically Correct site.

http://www.mathematicallycorrect.com

Solution: Answers will vary

Present your opinion of the Mathematically Sane site.

http://www.mathematicallysane.com/home.asp

Solution: Answers will vary

Compare and contrast the Mathematically Sane and Mathematically correct site positions. Where do you side, and why?

Solution: Answers will vary

Develop a position paper on whether or not there is value to spending so much time teaching students to deal manually with decimals in light of the existence of inexpensive technology that deals with them so easily.

Solution: Answers will vary. Some will probably say that calculators should not be used basically because they had to learn without calculators. If it was good enough for them, it is good enough for the students of today.

Some will say that calculators are the ruination of today’s students. They undoubtedly will cite the poor arithmetic performance of today’s students as evidence of their position. They will say students reflexively reach for a calculator when asked to do any arithmetic problem.

Certainly there is a need for students to memorize the arithmetic facts. They need to be functional with the algorithms for addition, subtraction, multiplication, and division. But, there has to be a time when technology is permitted. Few argue the need for an individual to add ten 7-digit addends by hand, for example. Somewhere between the addition facts and the need to add ten 7-digit addends is a point where technology (the calculator) is acceptable. The real argument is, where is it acceptable. Most people will present a point based upon personal experience and opinion. Few will provide a research-founded response. Regretfully, that is not the best response for most students.

Consider the answer you developed for the preceding question. Would it be any different if you were talking to students, parents, members of your department, teachers who work in grades before yours and, finally, school administrators?

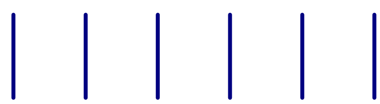
Why would you make any changes to deal with different groups or why would you leave things the same?

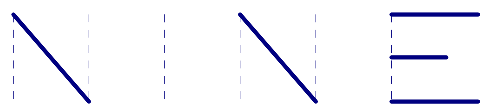
Solution: Answers will vary. Regretfully, most of the answers will be opinion-based, as opposed to research based. Furthermore, many responses will be influenced by what the writer thinks the audience wants to hear rather than what they need to hear.

Divide 30 by $\frac{1}{2}$ and add 10. What is the answer? Explain an error most students will make.

Solution: Most people who do this incorrectly will say that $30 \div \frac{1}{2} = 15$, rather than 60. Thus, they will say the answer is 25 instead of 70, which is correct.

Given six congruent parallel line segments arranged so they are perpendicular to an imaginary horizontal line, add five more line segments to make nine.



Solution: 

Where does the "Z" go and why?

A EF HI KLMN T VWXY (above the segment)

BCD G J OPQRS U (below the segment)

Solution: Below the segment holds all curves and above holds all straights, so the Z goes above the segment.

What value can be added to 1,000,000 so the result will be larger than if the 1,000,000 is multiplied by the same value? Is your answer unique? Why or why not?

Solution: Zero. No, the answer is not unique because adding 2 zeros would make the value even larger but multiplying by 2 zeros still yields a product of zero.

Given a nine-stall-long horse barn, how can ten horses be housed such that no two horses occupy the same stall, none are running free, none are being ridden, and so on.

T	E	N	H	0	R	S	E	S

Pick a number. Triple it. Add 12. Divide by 3. Subtract 4. What do you get? Why does this work?

Solution: 5	x
5 x 3 = 15	3x
15 + 12 = 27	3x + 12
$\frac{27}{3} = 9$	$\frac{3x + 12}{3} = x + 4$
9 – 4 = 5	x + 4 – 4 = x

You end up with the number you started with. Tripling is undone by dividing by 3. Adding 12 is undone by first dividing by 3 and then subtracting the residual 4.

Select a three-digit number (374). Affix a duplicate of that number to either end, giving a six-digit number (374,374). Divide the six-digit number by 7. Divide the missing factor from that division by 11. Divide the missing factor from that second division by 13. What did you get? Why does this work?

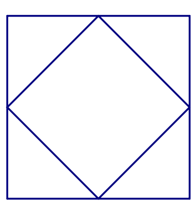
Solution: $\frac{374374}{7} = 53482$, $\frac{53482}{11} = 4862$, $\frac{4862}{13} = 374$. The fact is, (7)(11)(13) = 1001. In general, a 3-digit number xyz(1001) = xyz (1000 + 1) = xyz000 + xyz, but since 0 is the additive identity, you end up with xyzxyz. Dividing xyzxyz by 1001 myst leave xyz.

Some movie scripts describe ransom situations where a million dollars in small unmarked bills is to be left at some remote location. The alleged culprit picks up the package and runs away. Suppose the payment was in $10 bills. How much would the $1,000,000 weigh? A bill in United States currency weighs approximately 1 gram. What is a reasonable denomination for the alleged culprit to request so the escape can be effected?

Solution: $1,000,000 in $10 bills would mean you had 100,000 bills, each weighing 1 gram. So the stack would weigh 100 kilograms, which is about 220 pounds. Most people would not be able to run at all with that kind of weight, let alone go for any distance, which is what movie heroes often do. As far as a reasonable denomination, answers will vary.

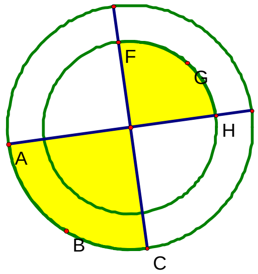
A window was a square, a meter on a side. That window admitted too much light so half the area was covered. After that, the window was still a meter high and a meter wide. How can that be?

Solution:



Use three line segments (straight, curved, open, or closed) to divide a circle into eight sections, each of which has the same area.

Solution:



The ratio of the long radius to the short radius is approximately 1.42:1.

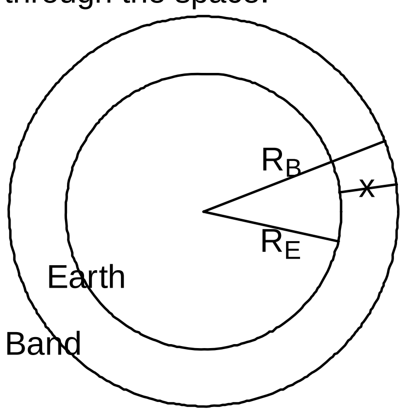
A bear was followed 3 miles south, 3 miles west, and then 3 miles north. At that point, the trail crossed the initial trail. What color was the bear?

Solution: White. The only place this can happen is if you start at the north pole.

Suppose the earth is a sphere with an equator 25,000 miles in length. A circular band is placed around the equator that is concentric with it and 25,000 miles plus 10 feet long. Which of the following most closely approximates the distance between the band and the equator:

(A) Thickness of standard piece of notebook paper
(B) 8.5-inch side of standard piece of notebook paper
(C) 11-inch side of standard piece of notebook paper
(D) You

Solution: You could fit through if you were to lie on the ground and squirm through the space.



R_E = Radius of the Earth

R_B = Radius of the Band

$x = R_B - R_E$

C_E = Circumference of the Earth

C_B = Circumference of the Band

C_E = 25,000 miles

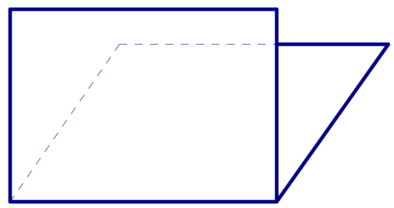
$C_B = 2\pi R_B$

$C_B = 2\pi(R_E + x)$

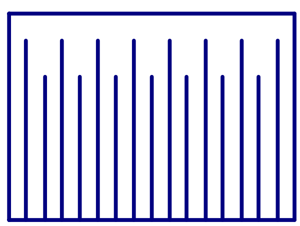
$C_B = 2\pi R_E + 2\pi x$

$C_B = C_E + 10$ feet
$C_B = 2\pi R_E + 10$ feet
So, $2\pi R_E + 2\pi x = 2\pi R_E + 10$ feet
$2\pi x = 10$ feet Subtracting $2\pi R_E$ from both sides
$x = \dfrac{5 \text{ feet}}{\pi} \approx 1.59$ feet

A “hole” can be cut in an 8.5-inch by 11-inch piece of paper that can be large enough for a person to step through. How?

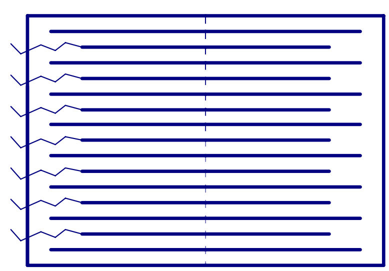


Solution: 1. Fold paper in half.
2. From the folded edge, make a series of cuts perpendicular to the fold
START WITH A LONG that goes almost all the way to open edges
THEN make a SHORT cut that does not go as close to open edges
ALTERNATE long, short, long, short, all the way across the paper
START and END with a LONG

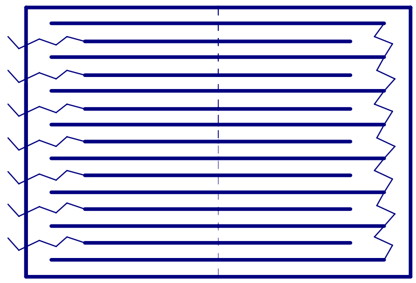


3. Open the paper
Starting at either edge, make collinear cuts to each short

3 indicates cut to short



4. At other edge
Cut long to long to long, making sure you miss the shorts between
DO NOT cut the edge



4 indicates cut long to long ...

Purchase a bag of M&Ms. Before opening it, predict the percentage of each color that will be present. Do you suppose the results from one bag are indicative of the all results? Why or why not?

Solution: Results will vary. Not all colors are equally represented and the number varies between bags. Check out http://us.mms.com/us/index.jsp for information on m&ms.

Investigate perfect numbers. In the process, answer questions like the ones that follow. You should not limit your research to answering the listed questions. What are the next two perfect numbers after 28? How many perfect numbers have been found to date? Has it been established that all perfect numbers have been found?

Solution: 496, 8128 Today 39 perfect numbers are known, ($2^{88}(2^{89}$- 1) is the largest). The jury is still out on whether there are more perfect numbers.

See http://mathworld.wolfram.com/PerfectNumber.html

Twelve, 18, 24, and 36 are listed as abundant numbers. Are all abundant numbers multiples of 6? Is there a multiple of 6 that is not an abundant number? Are there abundant numbers that are not multiples of 6? List examples and explain your position.

Solution: There are 21 abundant numbers less than 100, and they are all even. The first odd abundant number is 945 = $(3^3)(7)(5)$. The factors of 945 are 1, , 5, 7, 9, 15, 21, 27, 35, 45, 63, 105, 135, 189, 315, and ~~945~~ and, with 945 eliminated, the sum of the other factors is 975.

See http://mathworld.wolfram.com/AbundantNumber.html

Prime numbers are deficient numbers. Is there a pattern for nonprime deficient numbers? List examples and explain your position.

Solution: Primes, prime powers, and any divisors of a perfect or deficient number are all deficient. The first few deficient numbers are 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 19, 21, 22, 23, ...

See http://mathworld.wolfram.com/DeficientNumber.html

Resources

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Brumbaugh, D., Rock, D. (2006 (3rd Ed.). *Teaching Secondary Mathematics*. Mahwah, NJ: Lawrence Erlbaum Associates.

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