36 in = 1 yd
9 in = 0.25 yd (divide both sides by 4);
$\sqrt{9}$ in = $\sqrt{0.25}$ yd
3 in = 0.5 yd (positive square root of both sides).
Is it true that 3 inches equals a half a yard?
What is wrong with this "proof"?
Solution: No, 3 inches does not equal a half a yard. The trouble lies in expressing units in different representations.

Use the divisibility rule to show 3465 is divisible by 5.
Solution: 3465 = (346)(10) + 5
= (346)(5)(2) + 5
= 5[(346)(2) + 1]
= 5[692 + 1]
= 5[693] Note that unless the ones digit is a 0 or 5, a 5 could not be factored out of the sum and the original number would not be divisible by 5.

Use the divisibility to show 458952 is divisible by 8.
Solution: 458952 = (458)(1000) + 952
= (458)(125)(8) + (119)(8)
= 8[(458)(125) + 119)]
= 8[57250 + 119]
= 8[57369] Note that unless the last three digits are divisible by 8, a 8 could not be factored out of the sum and the original number would not be divisible by 8.

Prove the divisibility rule for 9 using VWXYZ where V, W, X, Y, and Z represent digits.
Solution: VWXYZ = 10000V + 1000W + 100 X + 10Y + Z
= (9999 + 1)V + (999 + 1)W + (99 + 1)X + (9 + 1)Y + Z
= 9999V + 999W + 99X + 9Y + V + W + X + Y + Z
Each of 9999V, 999W, 99X, 9Y is divisible by 9. If V + W + X + Y + Z is divisible by 9 the original number is and if not, the original number is not divisible by 9.

Prove the divisibility rule for 11 using UVWXYZ where U, V, W, X, Y, and Z represent digits.
Solution: UVWXYZ = $U(10)^5 + V(10)^4 + W(10)^3 + X(10)^2 + Y(10)^1 + Z(10)^0$
$= U(11 - 1)^5 + V(11 - 1)^4 + W(11 - 1)^3 + X(11 - 1)^2 + Y(11 - 1)^1 + Z(11 - 1)^0$.
Expanding each power term will give only one term that is not a factor of 11, and that will be the letter that is multiplied by the $(11 - 1)^n$ factor. For example, in the $U(11 - 1)^5$ expansion, the one term that would not have a factor of 11 would be "U". In all cases of $(11 - 1)^n$, where n is any counting number - - all terms except

for 1^n will be multiples of 11. The sign of 1^n will be negative for odd values of n, and positive for even ones. Thus, using UVWXYZ, you have ^-U, ^+V, ^-W, ^+X, ^-Y, and ^+Z, which can be expressed as subtracting the sums of alternated digits.

Over 300 proofs of the Pythagorean theorem have been listed (Loomis, 1963). Describe two Pythagorean proofs that are new to you.

Solution: Answers will vary.

Resources

Brumbaugh, D. K., Ortiz, E., Gresham, G. (2006). *Teaching Middle School Mathematics*. Mahwah, NJ: Lawrence Erlbaum Associates.

Brumbaugh, D., Rock, D. (2006 (3rd Ed.). *Teaching Secondary Mathematics*. Mahwah, NJ: Lawrence Erlbaum Associates.

Brumbaugh, D., Rock, D. (2001). *Scratch Your Brain C1*. Pacific Grove, CA: Critical Thinking Books and Software.