Unit 10 Quadratic Equations

Section 1 Explore Quadratic Graphs

Standard form of a Quadratic Function:

can be written in the form $y = ax^2 + bx + c$, where $a \neq 0$.

Examples: $y = 2x^2 - 3x + 2$

$y = 4x^2$ ----- $b = 0$ and $c = 0$

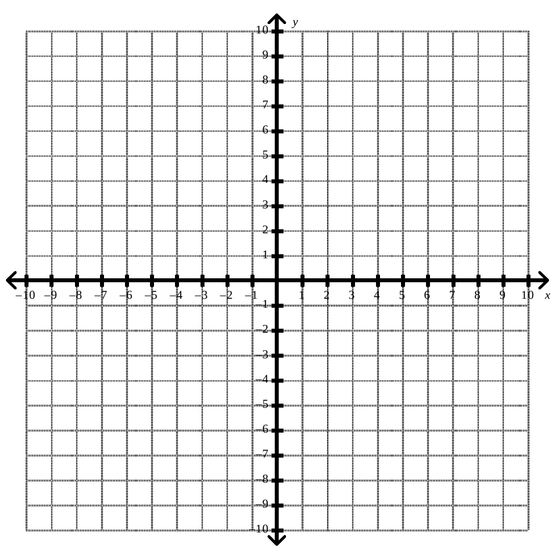
$y = x^2 - 5$ ----- $b = 0$

Example: Graph $y = x^2$

1st : fill in the table below:

x	$y = x^2$	y	(x,y)
-3	$(-3)^2$	9	(-3 , 9)
-2			
-1			
0			
1			
2			
3			

2nd : plot the ordered pairs on the graph below:



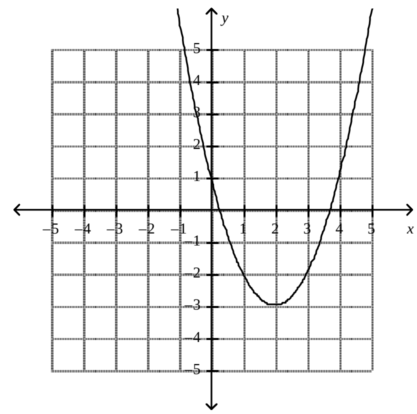
Does this graph remind you of any letter?

Unit 10 Quadratic Equations
Section 8-- Using the Discriminant

<u>Discriminant</u> – the expression under the radical sign in the quadratic formula, depending on the solution, it tells how many real solutions there are to a quadratic equation.

A Quadratic Equation has three types of Real Solutions

<u>Type I:</u> *<u>Two Real Solutions</u>*

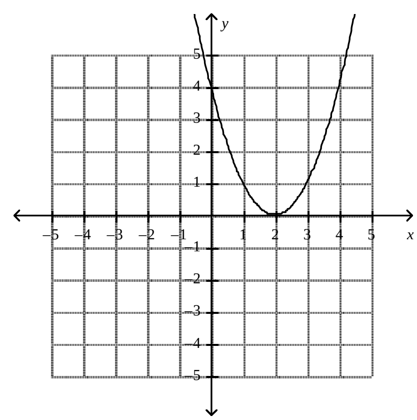


$y = x^2 – 4x + 1$
Find the values for a,b,c
a = 1; b = -4 ; c = 1
Find Discriminant:

$$\mathbf{b^2 – 4ac}$$
$$(-4)^2 – 4(1)(1)$$
$$16 - 4$$
$$12$$

We will conclude that if the Discriminant is greater than 0 there are 2 Real Solutions. $\mathbf{b^2 – 4ac > 0}$

<u>Type II:</u> *<u>One Real Solution</u>*



$y = x^2 – 4x + 4$
Find the values for a,b,c
a = 1; b = -4 ; c = 4

Find Discriminant:

$$\mathbf{b^2 – 4ac}$$
$$(-4)^2 – 4(1)(4)$$
$$16 – 16$$
$$0$$

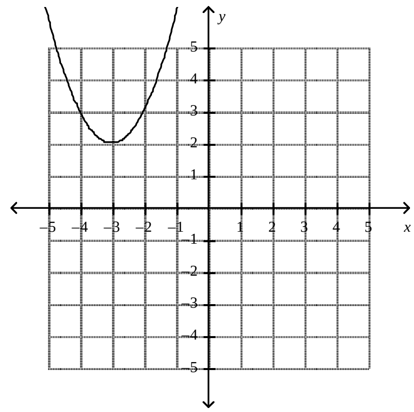
We will conclude that if the Discriminant is equal to 0 there is 1 Real Solution.
$\mathbf{b^2 – 4ac = 0}$

Type III: No Real Solution

$y = x^2 + 6x + 11$

Find the values for a,b,c

a = 1; b = 6 ; c = 11



Find Discriminant:

$$\mathbf{b^2 - 4ac}$$
$$(6)^2 - 4(1)(11)$$
$$36 - 44$$
$$-8$$

We will conclude that if the Discriminant is less than 0 there is NO Real Solution.

$$\mathbf{b^2 - 4ac < 0}$$

Now You Try!!

Find the Discriminant and tell how many real solutions for the given quadratic equation.

1) $x^2 - 10x + 3 = 0$

2) $3x^2 = 27$

3) $x^2 = 2x - 3$

4) $x^2 + 10x = -25$

5) $x^2 - 6x + 7 = 0$

6) $2x^2 - 8x = 4$

7) $3x + 3 = -x^2$

8) $8x^2 - 5x = 7$

Key:

1) 99; Two Real Solutions
2) –324; No Real Solutions
3) –8; No Real Solutions
4) 0; One Real Solution
5) 8; Two Real Solutions
6) 96; Two Real Solutions
7) –3;No Real Solution
8) –199; No Real Solution

Unit 10 Quadratic Equations
Section 7A-- Using Quadratic Formula

Quadratic Formula – Will solve any quadratic equation

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The variables are from standard form of a quadratic equation:

$$\mathbf{ax^2 + bx + c}$$

<u>Using the Quadratic Formula</u>

1. $x^2 - 4x + 3 = 0$ — Step 1: Identify a, b and c. a = 1, b = -4, c = 3

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Step 2: Write Quadratic Formula

$$\frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(3)}}{2(1)}$$

Step 3: Fill in the variables (Plug) (Substitution)

$$\frac{4 \pm \sqrt{16 - 12}}{2}$$

Step 4: Evaluate the expression (Chug) Solve using Order of Operations

$$\frac{4 \pm \sqrt{4}}{2}$$

$$\frac{4+2}{2}, \frac{4-2}{2}$$

Step 5: Split the $\pm$ into two expressions and Solve

3 , 1

Step 6: Check solutions. Substitute roots back into original quadratic equation.

$x^2 - 4x + 3 = 0$	$x^2 - 4x + 3 = 0$
$3^2 - 4(3) + 3 = 0$	$1^2 - 4(1) + 3 = 0$
$9 - 12 + 3 = 0$	$1 - 4 + 3 = 0$
0 = 0 √	0 = 0 √

The roots are 3 and 1

2. $2c^2 + 14c = 10$ Step 1: Put the equation in standard form

$\quad\quad\quad -10 \quad -10$ The equation should equal 0

$2c^2 + 14c - 10 = 0$ Step 2: Follow the steps in above problem

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(14) \pm \sqrt{(14)^2 - 4(2)(-10)}}{2(2)}$$

$$\frac{-14 \pm \sqrt{196 + 80}}{4}$$

$$\frac{-14 \pm \sqrt{276}}{4}$$

$$\frac{-14 \pm 4\sqrt{69}}{4}$$

$$\frac{-7 \pm \sqrt{69}}{2}$$

Irrational Solutions can be left in this form

Or

$$\frac{-7 + \sqrt{69}}{2}, \quad \frac{-7 - \sqrt{69}}{2}$$

Approximate (≈) Solutions using decimals

≈ 0.65 , -7.653

Now you try!!

Solve by using Quadratic Equations. Round answers to the nearest tenth if necessary.

1) $x^2 + 7x + 6 = 0$

2) $-x^2 + 6x - 5 = 0$

Hint: It does not matter what the letter is as long as it looks like a quadratic.

3) $3w^2 + 5w - 8 = 0$

4) $4n^2 + 8n - 3 = 0$

5) $2t^2 + 4t = 3$

6) $8k^2 - 5k = 7$

Section 7B Using the Appropriate Method

You have learned several ways to solve a quadratic equation. Below is a table that reviews these methods.

Method	When is the Method Used
Graphing	Use only to estimate solutions
Using Square Roots	When the Quadratic equation is or can be put into the form $x^2 = a$
Factoring	Use when the Quadratic expression is easy to factor
Completing the Square	Use when a = 1 and all other coefficients are fairly small
Quadratic Formula	Use on any quadratic equation

Give the method(s) you would choose to solve each equation and explain why. (Only pick Quadratic Formula once)

7) $4x^2 - 64 = 0$

8) $x^2 - 4x = 21$

9) $3x^2 - 11x - 2 = 0$

10) $x^2 + 4x - 7 = 0$

Use any method you choose to solve each equation. If necessary, round to the nearest tenth. (You must use at least 3 different methods)

11) $x^2 - 36 = 0$

12) $4x^2 - 52x + 133 = 0$

13) $x^2 + 13x - 30 = 0$

14) $8x^2 + 25x + 19 = 0$

15) $x^2 + 6x + 5 = 0$

16) $5x^2 - 20 = 0$

Key:

1) 6,1
2) 1,5
3) $\frac{58}{3}, \frac{-8}{3}$
4) $\frac{-2 \pm \sqrt{7}}{2}$
5) $\frac{-2 \pm \sqrt{10}}{2}$
6) $\frac{5 \pm \sqrt{1049}}{16}$
7) Square Roots $x^2 = 16$ or Factoring 4(x+4)(x-4)
8) Factoring (x – 7)(x + 3) or Quadratic Formula or Completing Square add 4 to each side.
9) Quadratic Formula
10) Completing the Square add 4 to each side
11) ± 6
12) $\frac{19}{26}, \frac{7}{26}$
13) (x+5)(x-2) = 0 ; x = -5, 2
14) $\frac{1}{2}, \frac{-21}{8}$
15) (x +5)(x+1) = 0 ; x =-5,-1
16) $x^2 = 4$; $x = \pm 2$

Unit 10 Quadratic Equations

Section 6-- Completing the Square

Remember how to FOIL:

$$(x + 3)(x + 3)$$

F + O + I + L

$$(x \bullet x) + (x \bullet 3) + (3 \bullet x) + (3 \bullet 3)$$

$$x^2 + 3x + 3x + 9$$

$$x^2 + 6x + 9$$

$x^2 + 6x + 9$ is a perfect square trinomial because when you factor it ("unfoil") you get two of the same binomials = $(x + 3)(x + 3) = (x + 3)^2$

An expression ($x^2 + bx + c$) can be changed into a perfect square trinomial.

1. Find c if $x^2 + 6x + c$ is a perfect square trinomial

Make the "b" list: $b = 6$

$$\frac{b}{2} = \frac{6}{2} = 3$$

$$3^2 = 9$$

Therefore c = 9 so the perfect square trinomial is $x^2 + 6x + 9 = (x + 3)^2$

2. Find the value for m so that the trinomial is a perfect square trinomial

$x^2 - 9x + m$ $\quad b = -9$

$$\frac{b}{2} = \frac{-9}{2}$$

$$\left(\frac{-9}{2}\right)^2 = \frac{81}{4}$$

Therefore c = $\frac{81}{4}$ so the perfect square trinomial is $x^2 - 9x + \frac{81}{4} = \left(x - \frac{9}{2}\right)^2$

Solving Equations by completing the square:

1\. $x^2 + 10x = 24$ Step 1. Make the "b" list: b= 10 b/2 = 10/2 = 5, $5^2 = 25$

$+25 \quad +25$

Step 2: Add 25 to each side

$x^2 + 10x + 25 = 49$

$(x + 5)^2 = 49$ Step 3: Factor the complete square

$\sqrt{(x+5)^2} = \sqrt{49}$ Step 4: "un"square each side by taking the square root of each side

$(x + 5) = \pm 7$

$x + 5 = 7$ or $x + 5 = -7$ Step 5: Set equal to each solution & solve for x

$-5 \quad -5 \qquad -5 \quad -5$

$x = 2$ or $x = -12$ Step 6: Check each in original equation.

$2^2 + 10(2) = 24$ √ $(-12)^2 + 10(-12) = 24$√

2\. $a^2 + 3a = 0$ b=3

$+\frac{9}{4} \quad +\frac{9}{4}$ b/2= 3/2

$a^2 + 3a + \frac{9}{4} = \frac{9}{4}$ $(3/2)^2 = 9/4$

$(a + \frac{3}{2})^2 = \frac{9}{4}$

$\sqrt{(a + \frac{3}{2})^2} = \sqrt{\frac{9}{4}}$

$a + \frac{3}{2} = \pm\frac{3}{2}$

$a + \frac{3}{2} = \frac{3}{2} \qquad a + \frac{3}{2} = -\frac{3}{2}$

$-\frac{3}{2} \quad -\frac{3}{2} \qquad -\frac{3}{2} \quad -\frac{3}{2}$

a = 0 or a = -3

3\. $w^2 + 22w + 85 = 0$ b = 22

$-85 \quad -85$ b/2=22/2=11

$w^2 + 22w = -85$

$11^2 = 121$

$+121 \quad +121$

$w^2 + 22w + 121 = 36$

$(w + 11)^2 = 36$

$\sqrt{(w+11)^2} = \sqrt{36}$

$w + 11 = \pm 6$

$w + 11 = 6 \qquad w + 11 = -6$

$-11 \quad -11 \qquad -11 \quad -11$

w = -5 or w = -17

When you have a value for a ($ax^2 + bx + c$), first you divide both sides by a.

$\frac{3a^2 - 18a}{3} = \frac{30}{3}$ Divide by 3 on both sides

$a^2 - 6a \quad = 10$ b=6

$\underline{\qquad +9 \quad +9}$ b/2=6/2=3

$a^2 - 6a + 9 = 19$ $3^2 = 9$

$(a - 3)^2 = 19$

$\sqrt{(a-3)^2} = \sqrt{19}$

$a - 3 = \pm\sqrt{19}$

$\underline{\quad +3 \quad +3}$

$\boxed{a = 3 \pm \sqrt{19}}$

Now Try These!!

Find the value d in order to make each trinomial a perfect square trinomial.

1) $k^2 + 14k + d$

2) $s^2 - 2s + d$

3) $z^2 - 10z + d$

4) $m^2 - m + d$

Solve each equation by completing the square

5) $x^2 - 8x = 7$

6) $r^2 + 6r = -5$

7) $n^2 - 2n - 30 = 0$

8) $s^2 - 6s - 4 = 0$

9) $p^2 + 8p + 9 = 0$

10) $3a^2 - 12a = 24$

Answer Key 10.6

1) 49

2) 1

3) 25

4) $\frac{1}{4}$

5) 1,7

6) –5, -1

7) $1 \pm \sqrt{31}$

8) $3 \pm \sqrt{13}$

9) $-4 \pm \sqrt{7}$

10) 5, -1

Unit 10 Quadratic Equations
Section 5 Factoring to Solve

Zero Product Property

For every real number a and b if ab = 0 then a = 0 or b = 0.

In other words, if you are multiplying two or more factors together and the product is 0, then at least one of the factors must be 0.

Example 1: $xy = 0$

Set each factor (the things being multiplied) equal to 0.

$x = 0$ or $y = 0$

Either x or y has to be 0 in order for their product to equal 0.

Example 2: $(x - 2)(3x + 1) = 0$

Set each of the factors equal to 0.

$$x - 2 = 0 \qquad 3x + 1 = 0$$
$$\underline{+2 \quad +2} \qquad \underline{-1 \quad -1}$$
$$x = 2 \qquad \frac{3x}{3} = \frac{-1}{3}$$
$$x = \frac{-1}{3}$$

Check Solutions: $(2-2)(3(2) + 1) = 0$ $\qquad (-\frac{1}{3} - 2)(3(\frac{-1}{3} + 1) = 0$

$(0)(7) = 0$ √ $\qquad (-2\frac{1}{3})(0) = 0$ √

So $x = 2$ or $\frac{-1}{3}$

Solving Quadratics that Factor

You will want to review factoring before you begin this part of the lesson.

Go back to Unit 9, sections 4, 5 and 6.

How to:

1. Factor the quadratic equation
2. Set each factor equal to 0
3. Solve for x in each equation
4. Check solutions with original problem
5. Give solutions

Examples:

1. $x^2 - 7x + 6 = 0$ — Quadratic Equation form $x^2 + bx + c$

$(x - 6)(x - 1) = 0$ — Factoring (You can check by FOIL-ing)

$x - 6 = 0 \quad x - 1 = 0$ — Set each factor equal to 0

$\underline{\quad +6 \quad +6} \quad \underline{\quad +1 \quad +1}$ — Solve each Equation

$x = 6$ or $x = 1$

$6^2 - 7(6) + 6 = 0$ — $0 = 0$ √ — $1^2 - 7(1) + 6 = 0$ — $0 = 0$ √ — Check solutions. Both Solutions work.

Therefore x = 6 or 1

2. $3x^2 - 2x - 21 = 0$ — Quadratic Equation form $ax^2 + bx + c$

$(3x + 7)(x - 3) = 0$ — Factor (You can check by FOIL-ing)

$3x + 7 = 0 \quad x - 3 = 0$ — Set each factor equal to 0

$\underline{\quad -7 \quad -7} \quad \underline{\quad +3 \quad +3}$ — Solve each Equation

$\frac{3x}{3} = \frac{-7}{3} \quad x = 3$

$x = \frac{-7}{3}$

$3\left(\frac{-7}{3}\right)^2 - 2\left(\frac{-7}{3}\right) - 21 = 0$ — $0 = 0$√ — $3(3)^2 - 2(3) - 21 = 0$ — $0 = 0$√ — Check solutions. Both solutions work!

Therefore $x = \frac{-7}{3}$ or 3

Now you try these!!

Use Zero Product Property to solve each equation.

1) $-2n(4n - 3) = 0$

2) $(x + 4)(6x + 11) = 0$

3) $a(a - 2) = 0$

4) $(z + 8)(z - 2) = 0$

Solve by Factoring

5) $x^2 + 10x + 16 = 0$

6) $k^2 - 5x - 6 = 0$

7) $9w^2 + 24w + 16 = 0$

8) $4d^2 - 10d + 6 = 0$

9) $t^2 - 7t = 0$

10) $r^2 = -14r$ (Hint: make it equal to 0)

Key

1) 0, $\frac{3}{4}$

2) –4, $\frac{-11}{6}$

3) 0, 2

4) –8, 2

5) (x + 8)(x + 2) = 0
   x = -8, -2

6) (k – 6)(k + 1) = 0
   K = 6, -1

7) (3w – 4)(3w – 4) = 0
   w = $\frac{4}{3}$

8) (4d – 6)(d – 1) = 0
   d = $\frac{3}{2}$, 1

9) t (t –7) = 0
   t = 0, 7

10) r^2 + 14r = 0
   r(r+ 14) = 0
   r = 0, -14

Unit 10 Quadratic Equations
Section 4 Solving Quadratic Equations

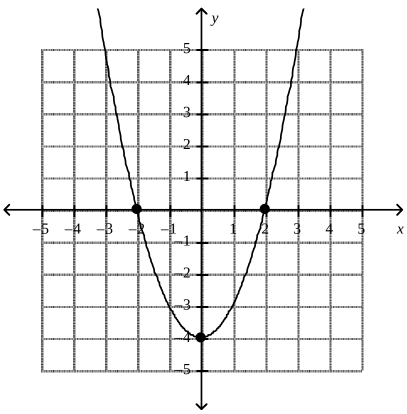
Solve by Graphing:

Solving a Quadratic Equation means finding the value(s) for x (the x intercepts).

Types of Real Number Solutions

I. Two Solutions

$y = x^2 - 4$ Graph the parabola and look for x – intercepts.



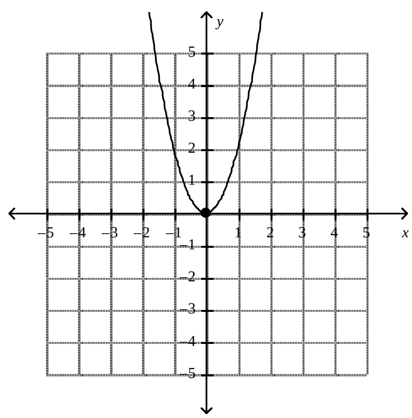
The solutions are +2 and –2**.**

This can be written x = ± 2.

II. One Solution

Graph the equation $y = 2x^2$.

The solution is x = 0.

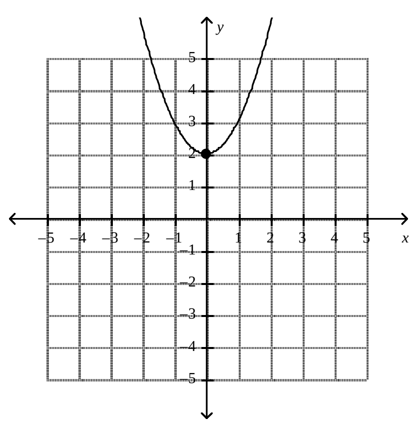


III. No Solution

Graph $y = x^2 + 2$.

The graph does not touch or cross the x – axis.

The solution is no solution.



Solve by Using Square Roots:

$0 = 2x^2 - 8$ Step 1: Isolate the variable on one side of the equation.

$+8 \quad +8$ Add 8

$\frac{8}{2} = \frac{2x^2}{2}$ Divide by 2

$4 = x^2$ Step 2: In order to "undo" a square you must take the square root of both sides of the equation.

$\sqrt{4} = \sqrt{x^2}$

$\boxed{\pm 2 = x}$ Step 3: Write the solution.

Other Examples:

a) $7n^2 + 6 = 6$

$-6 \quad -6$

$\frac{7n^2}{7} = \frac{0}{7}$

$\sqrt{n^2} = \sqrt{0}$

$n = 0$

b) $3a^2 - 12 = 0$

$+12 \quad +12$

$\frac{3a^2}{3} = \frac{-12}{3}$

$\sqrt{a^2} = \sqrt{-4}$

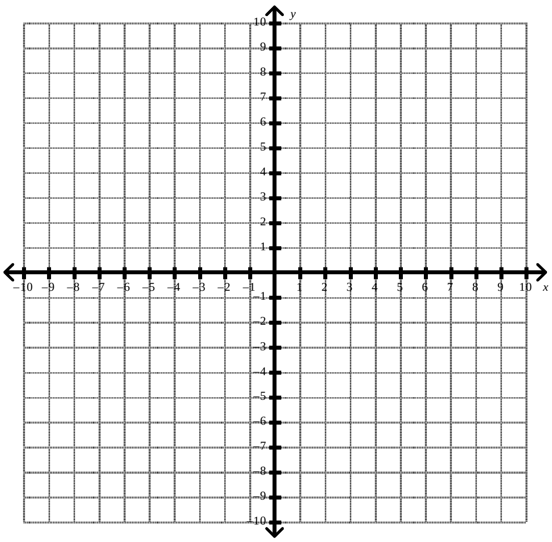
$a = \text{undefined}$

No Solution

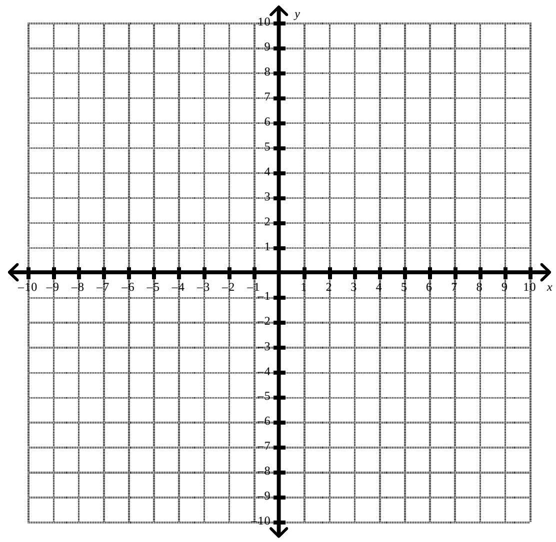
Now Try!!!

I. Solve by graphing.

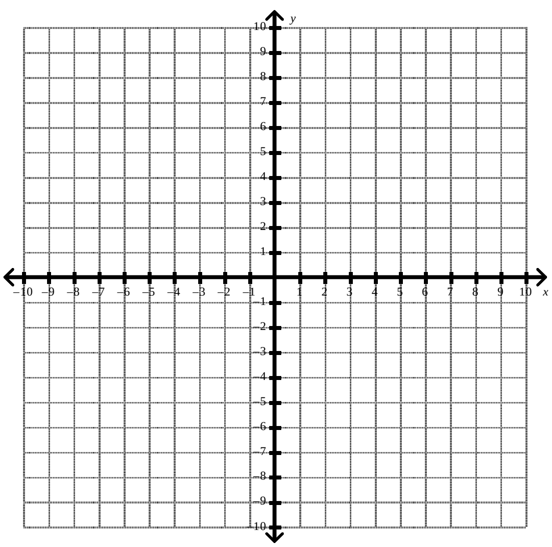
1. $y = x^2 + 9$



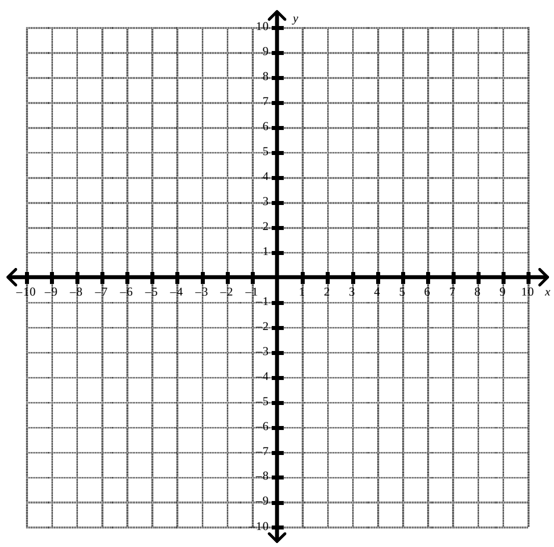
2. $y = \frac{1}{2}x^2$



3. $y = -x^2 - 4$



4. $y = -x^2 + 4$



Solve each equation.

5. $k^2 = 49$

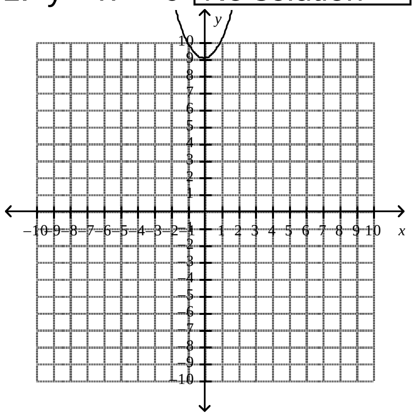
6. $7w^2 - 28 = 0$

7. $9r^2 = 16$

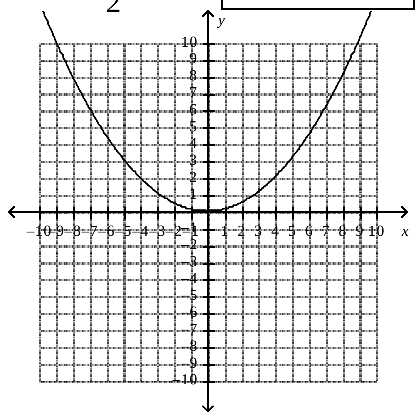
8. $a^2 + 25 = 25$

I. Solve by graphing.

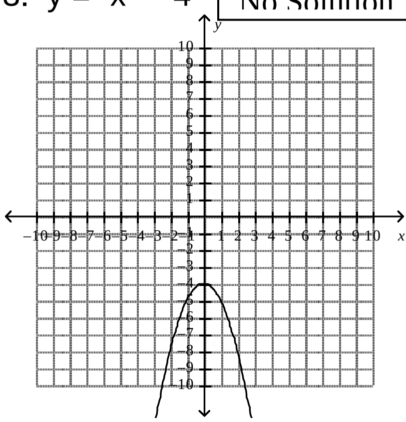
1. $y = x^2 + 9$ No solution



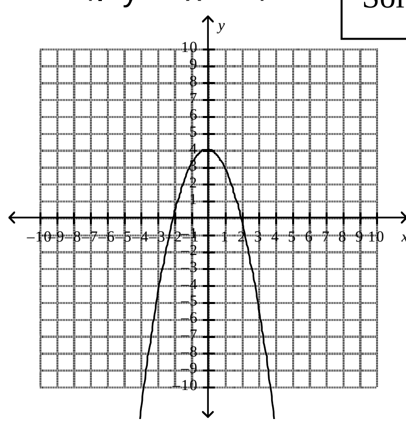
2. $y = \frac{1}{2}x^2$ Solution $x = 0$



3. $y = -x^2 - 4$ No Solution



4. $y = -x^2 + 4$ Solution $x = \pm 2$



Solve each equation.

5. $k^2 = 49$

$k = \pm 7$

7. $9r^2 = 16$

$r = \pm\frac{4}{3}$

6. $7w^2 - 28 = 0$

$w = \pm 2$

8. $a^2 + 25 = 25$

$a = 0$

Unit 10 Quadratic Equations
Section 3 Square Roots

<u>Square Root</u>- number a is a square root of b if $a^2 = b$

Example: 4 is a square root of 16 because $4^2 = 16$.
-4 is a square root of 16 because $(-4)^2 = 16$.
Therefore the square root of 16 is ± 4. The $\pm$ sign indicates both the square root.

<u>Radical sign</u> – the symbol that indicates a square root. $\sqrt{}$

$\sqrt{25}$ means the positive or <u>principal root</u> of 25 = 5
$-\sqrt{25}$ means the negative square root of 25 = -5

$\sqrt{25 \text{ is the radicand}}$ The <u>radicand</u> is the number under the radical sign.

<u>Simplifying Square Roots</u>

a) $\sqrt{64} = 8$ b) $-\sqrt{144} = -12$ c) $\sqrt{\frac{16}{256}} = \frac{\sqrt{16}}{\sqrt{256}} = \frac{4}{16} = \frac{1}{4}$

d) $\sqrt{0} = 0$ e) $\sqrt{-100}$ = undefined for the set of Real Numbers

<u>Rational or Irrational Square Roots:</u>

Reminder:
- Rational numbers can be expressed in the form $\frac{a}{b}$ where *a* and *b* are integers and $b \neq 0$. These include terminating (end) and repeating decimals.
- Irrational numbers cannot be expressed in the form $\frac{a}{b}$ where *a* and *b* are integers and $b \neq 0$. These include non terminating and non - repeating decimals.

Use a 4-function calculator to help!!!

Identify each root as rational or irrational.

Examples:

1) $\sqrt{9}$ -Press 9 then the $\sqrt{\ }$ keys the result should be $\pm$3.--- Rational
2) $\sqrt{2.56}$ = $\pm$1.6 – Rational
3) $\sqrt{7}$ = $\pm$2.6457513… ---- Irrational
4) $\sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5}$ ----- Rational
5) $\sqrt{\frac{1}{10}} = \frac{1}{3.162276}$ = .3162277… ------- Irrational

Estimating Square Roots to the nearest whole number.

Use a calculator and then round to the nearest whole number.

Examples:
1) $\sqrt{12}$ = 3.4… = 3   2) $\sqrt{24}$ = 4.8… = 5

Approximate the Square Roots to the nearest hundredths.
Use a calculator and round to the nearest hundredths.

0.X<u>Y</u>Z   Y is in the hundredths place
Use Z to round Y.

1) $\sqrt{171.9}$ = 13.1<u>1</u>1 = 13.11   2) $\sqrt{54.9}$ = 7.4<u>0</u>9 = 7.41

Simplify:

1) $-\sqrt{1024}$

2) $\sqrt{\frac{0.09}{0.16}}$

3) $\sqrt{\frac{81}{64}}$

4) $-\sqrt{676}$

5) $\sqrt{0.0036}$

6) $\sqrt{\frac{36}{196}}$

Tell whether each expression is rational or irrational.

7) $\sqrt{37}$

8) $\pm\sqrt{\frac{1}{9}}$

9) $\pm\sqrt{0.04}$

10) $\sqrt{14.4}$

Estimate each square root to the nearest whole number.

11) $\sqrt{131.4}$

12) $\sqrt{410}$

13) $\sqrt{2.314}$

14) $\sqrt{7}$

Approximate each square root to the nearest hundredth.

15) $\sqrt{18}$

16) $-\sqrt{273}$

17) $\sqrt{14{,}572}$

18) $-\sqrt{0.003}$

Key:

1. 32
2. .75
3. $\frac{9}{8}$
4. 26
5. .06
6. $\frac{3}{7}$
7. Irrational
8. Rational
9. Rational
10. Irrational
11. 11
12. 20
13. 2
14. 3
15. 4.24
16. –16.52
17. 3.82
18. –0.05

Unit 10 Quadratic Equations
Section 2 Quadratic Functions

Standard form of a Quadratic Function:
can be written in the form $y = ax^2 + bx + c$, where $a \neq 0$.

We already L☺☺Ked at $y = ax^2$ and $y = ax^2 + c$.

An equation with a value for *b* is a bit more complicated.

L☺☺K at **Quadratic Equations in the form $y = ax^2 + bx + c$.**

<u>Axis of Symmetry</u> – To find this we use the formula **$x = \frac{-b}{2a}$**

<u>Vertex</u> – Since the x - coordinate is the same as the value found for the axis of symmetry then we simply plug the x-value into our equation to arrive at our y – coordinate. $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

Example: Find the vertex and axis of symmetry without graphing.

1. $y = 2x^2 - 4x + 3$ Remember a = 2, b = (- 4), c = 3

$x = \frac{-b}{2a} = \frac{-(-4)}{2(2)} = \frac{4}{4} = 1$

The axis of symmetry is x = 1

We know the x-coordinate is 1 of the vertex. We need to find the y-coordinate.

$y = 2(1)^2 - 4(1) + 3$
$y = 2(1) - 4 + 3$
$y = 2 - 4 + 3$
$y = 1$

The Vertex is (1,1)

Using the previous process it is very easy to graph these types of Quadratic Equations.

<u>Graphing Quadratic Equations in the Form y = $ax^2 + bx + c$.</u>

Step 1: Find axis of symmetry.
Step 2: Find Vertex
Step 3: Graph Vertex
Step 4: Pick a value for x other than the one found.
Step 5: Plug in and get y – value for Step 4
Step 6: Plot Vertex and other point
Step 7: Reflect the other point over the axis of symmetry.
Step 8: Draw a Parabola

<u>Example:</u>

2. $f(x) = x^2 + 4x + 3$

Step 1:

$x = \frac{-b}{2a} = \frac{-4}{2(1)} = \frac{-4}{2} = -2$ axis of symmetry $x = -2$

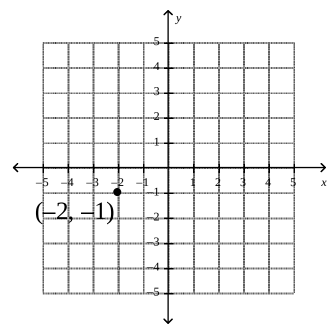
Step 2:

$y = (-2)^2 + 4(-2) + 3$ Vertex $(-2, -1)$

$y = 4 - 8 + 3$

$y = -1$

Step 3:

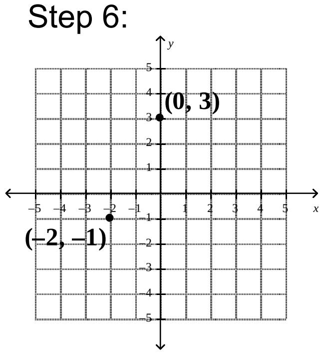


Step 4: x = 0 (I use this a lot of the time, makes for Easy math.)

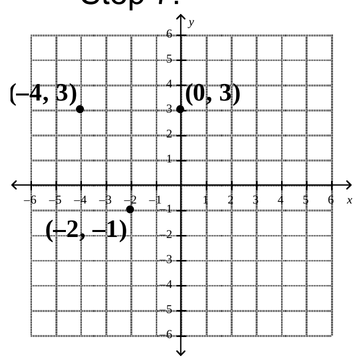
Step 5: $y = (0)^2 + 4(0) + 3$

$y = 3$ Other point (0,3)

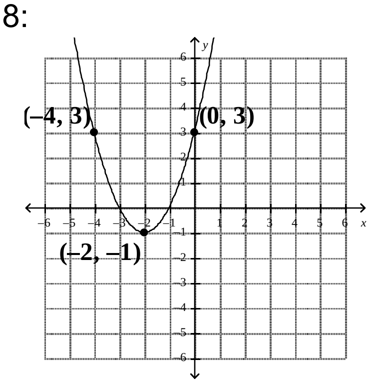
Step 6:



Step 7:



Step 8:



Graphing Quadratic Inequalities

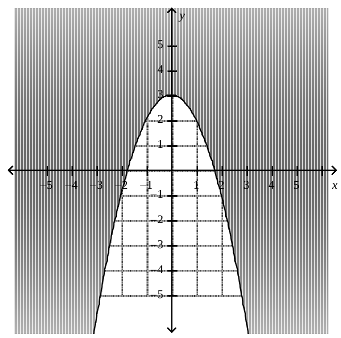
Step 1: Graph parabola – use a solid curve for $\leq$ or $\geq$

use a dotted curve for $<$ or $>$

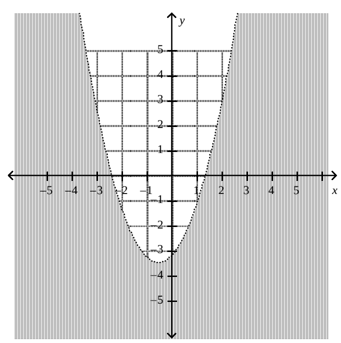
Step 2: Shade above for $>$ or $\geq$; Shade below for $<$ or $\leq$

Examples:

3. $f(x) \geq -x^2 + 3$



4. $y < x^2 + x - 3$

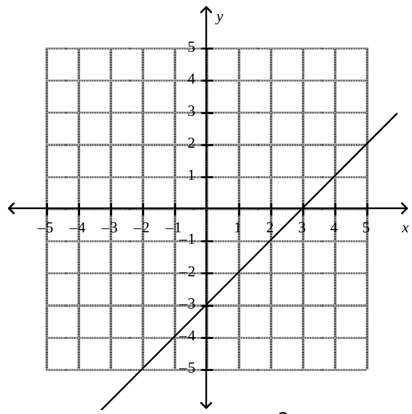


Unit 10 Quadratic Equations
Section 9-- Choosing A Model

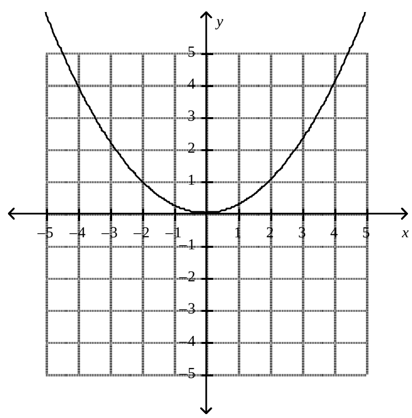
What will it be Linear, Quadratic or Exponential?

General Appearances

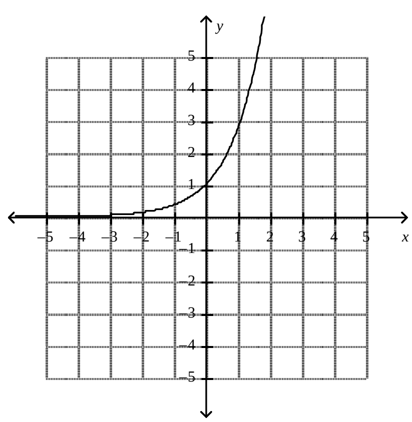
Linear – (y = mx + b) Points form a line



Quadratic – ($y = ax^2 + bx + c$) Points form a parabola (U – shape)



Exponential – ($y= ab^x$) Points form a slight curve

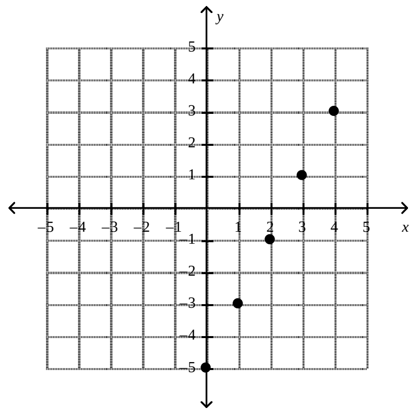


Graph each set of points and tell which model is most appropriate.

Examples

1.

x	y
0	-5
1	-3
2	-1
3	1
4	3

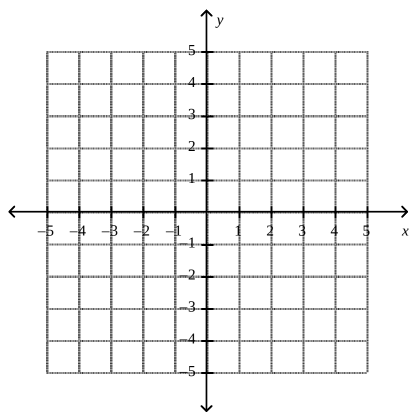


Linear

Now You Try!!

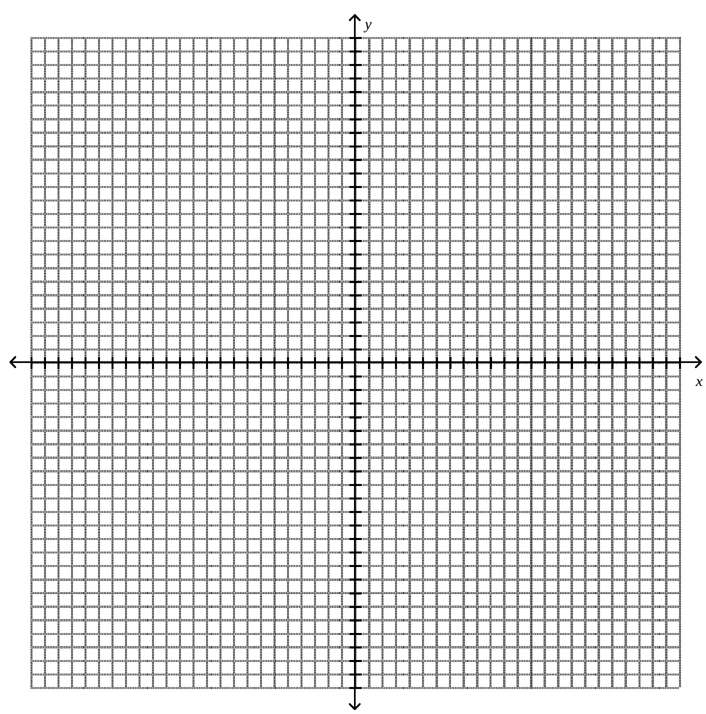
2.

x	y
0	5
1	2
2	0.8
3	0.32
4	0.128



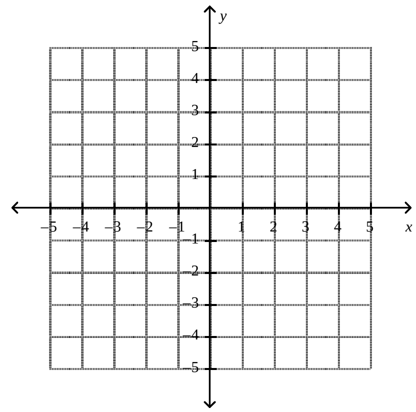
3.

x	y
0	0
1	1.5
2	6
3	13.5
4	24



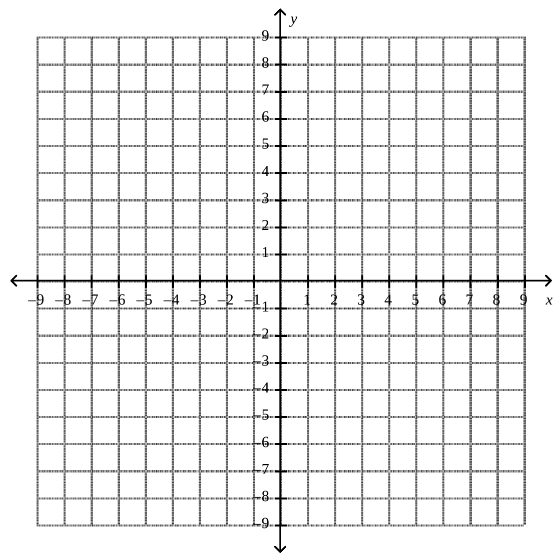
4.

x	y
0	2
1	1.5
2	1
3	0.5
4	0



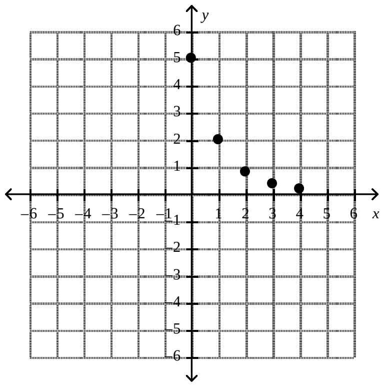
5.

x	y
0	1
1	1.5
2	2.5
3	4
4	8



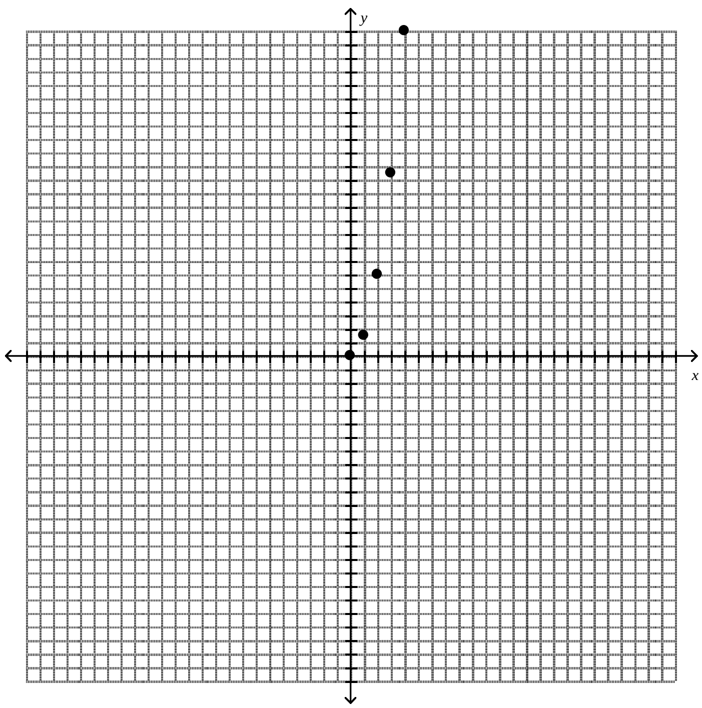
Key:

2)



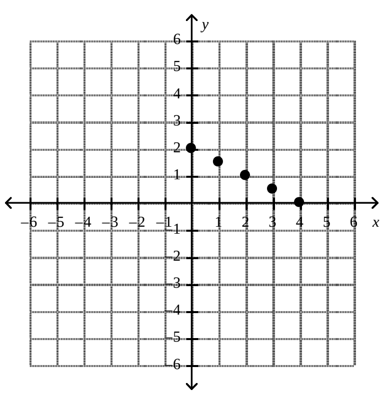
Exponential

3)

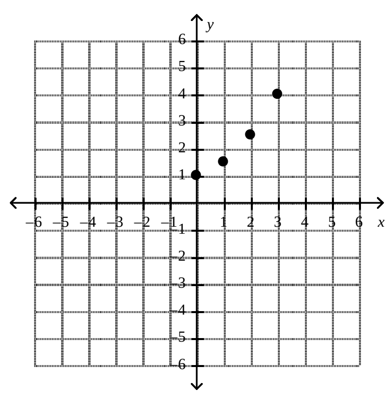


Quadratic

4)



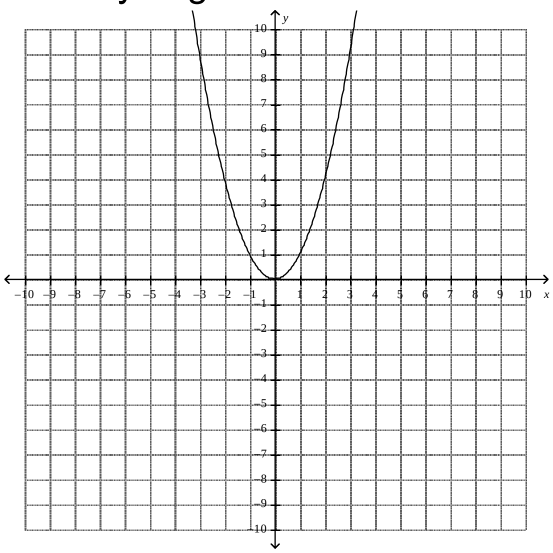
5)



Linear

Quadratic

Did you guess a U?? ------------- Not a V (remember y = |x|)



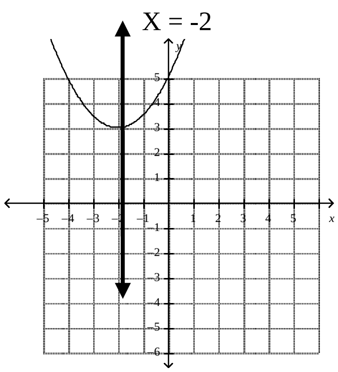
Parabola – The U shaped curve produced by a Quadratic Function.

Features of Parabolas

Vertex - The highest (maximum) or lowest (minimum) *point* of a parabola.

Axis of Symmetry – The *line* that divides the parabola into two matching halves. (Equation: x = {x-coordinate of the vertex})

Example: Identify the vertex and axis of symmetry of the graph.



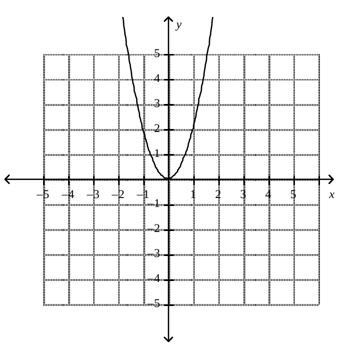
The vertex is at (-2,3).
The axis of symmetry is at x = -2.

Graphing Quadratics in the form y = *a*x^2

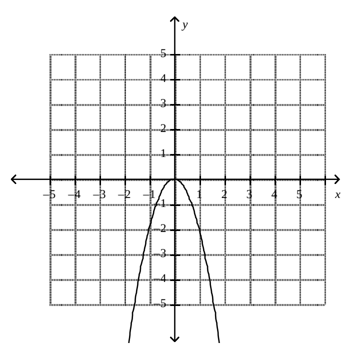
All graphs in this form have their vertex at (0,0).

L☺☺K AT THIS

$y = 2x^2$



$y = -2x^2$



Did you notice the difference between the equations and graphs?

In the graphs *a* = 2 or *a* = -2 respectively.

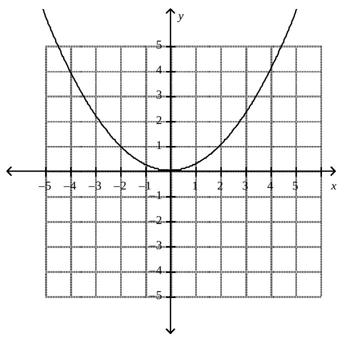
We can conclude from the graphs:

If **_a_ is positive the parabola opens up.**

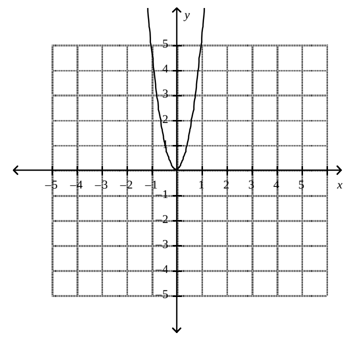
_a_ is negative the parabola opens down.

L☺☺K AT THIS

$y = \frac{1}{4}x^2$



$y = 5x^2$



Do you see what happened?

In the 1st graph a < 1 and the 2nd graph a > 1.

We can conclude from the graphs:

If **_a_ < 1 the parabola becomes wider (fatter).**

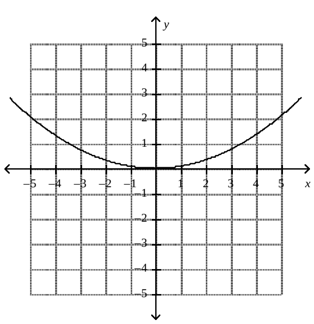
_a_ > 1 the parabola becomes more narrow (skinnier).

Graphing Quadratics in the form $y = ax^2 + c$

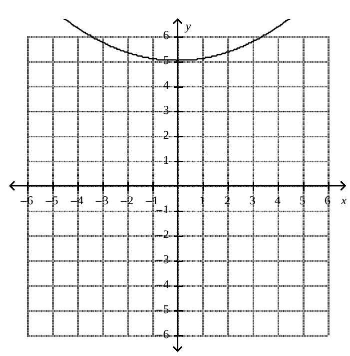
We are going to look at the effect of changing the value of *c*.

L☺☺K AT THIS

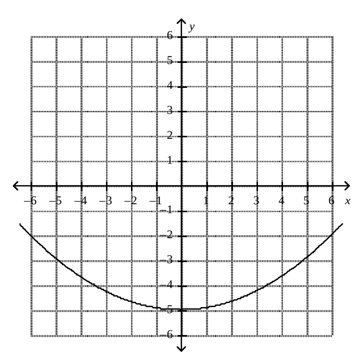
$y = \frac{1}{9}x^2$



$y = \frac{1}{9}x^2 + 5$



$y = \frac{1}{9}x^2 - 5$



Did you see that!! The parabola moved!!

What made it move? Well, the value for *c* makes quadratic equations in the form y = $ax^2 + c$ move up or down.

We can conclude from the graphs:
If +*c* the parabola moves up *c* units.
-*c* the parabola moves down *c* units.

Example: Without graphing describe how each graph differs from the graph of y = x^2.

$y = \frac{1}{2}x^2 + 4$ Opens up, Vertex (0,4) and wider than normal

$y = -2x^2 - 3$ Opens down, Vertex (0,-3) and narrower than normal

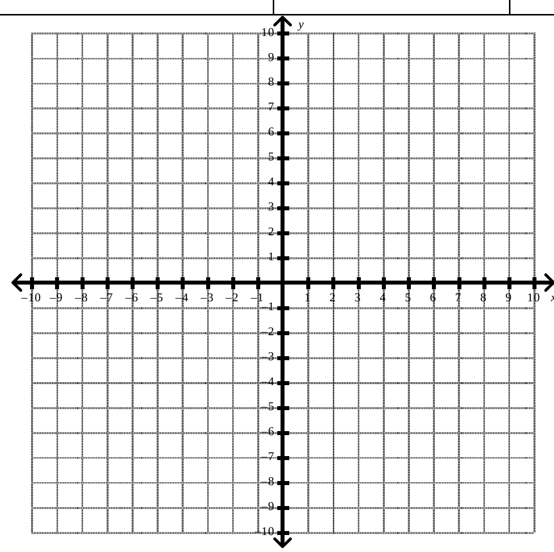
Chapter 10 / Section 1 Problems

Now you try!!

I. Use a table to graph.

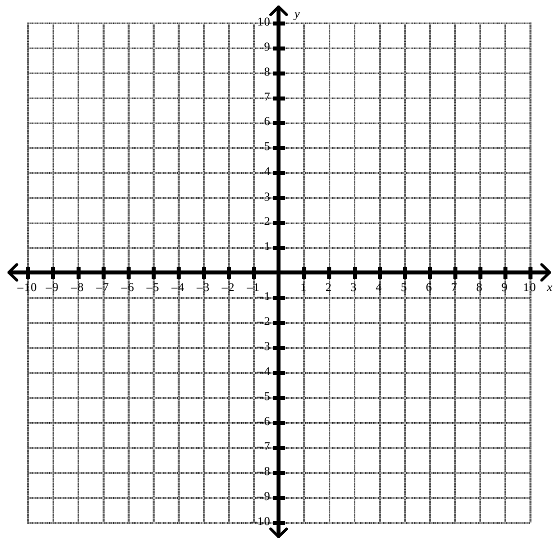
1.

x	$y = -2x^2$	y	(x,y)
-2			
-1			
0			
1			
2			



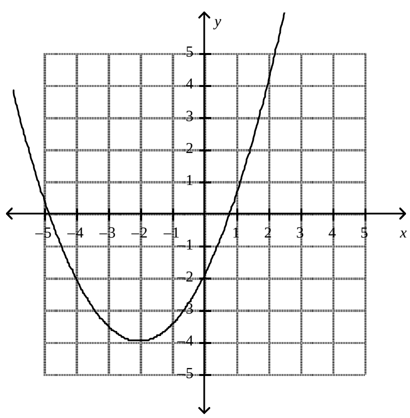
2.

x	$y = \frac{2}{3}x^2$	y	(x,y)
-6			
-3			
0			
3			
6			

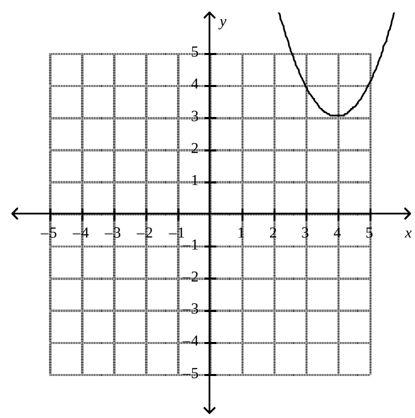


II. Identify the vertex and axis of symmetry of each graph.

3.



4.



III. Without graphing describe how each graph differs from the graph of $y = x^2$.

5. $y = -0.25x^2 + 5$

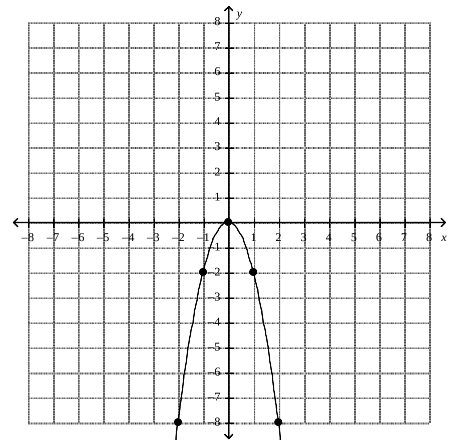
6. $y = 3x^2 - 5$

Answer Key:
Unit 10 / Section 1 Problems

Now you try!!

IV. Use a table to graph.

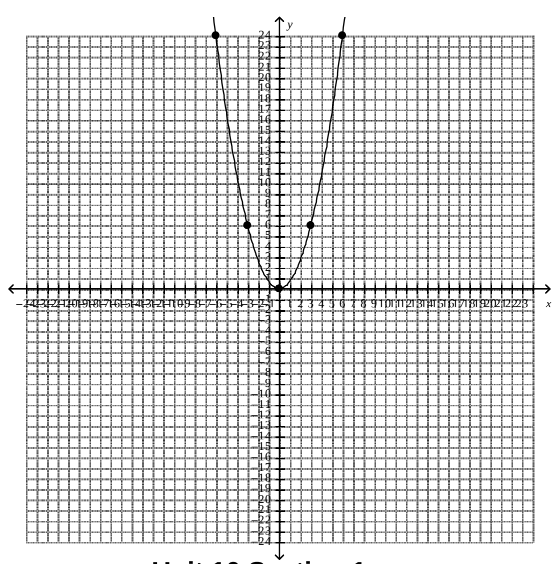
x	$y = -2x^2$	y	(x,y)
-2	$-2(-2)^2$	-8	(-2,-8)
-1	$-2(-1)^2$	-2	(-1,-2)
0	$-2(0)^2$	0	(0,0)
1	$-2(1)^2$	-2	(1,-2)
2	$-2(2)^2$	-8	(2,-8)



1.

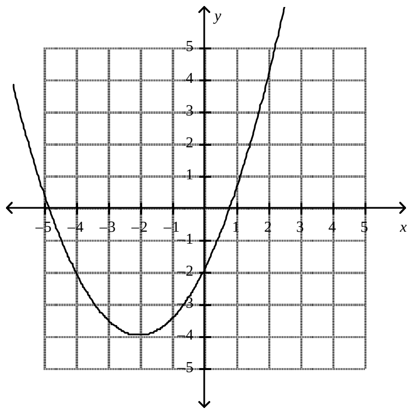
2.

x	$y = \frac{2}{3}x^2$	y	(x,y)
-6	$\frac{2}{3}(-6)^2$	24	(-6,24)
-3	$\frac{2}{3}(-3)^2$	6	(-3,6)
0	$\frac{2}{3}(0)^2$	0	(0,0)
3	$\frac{2}{3}(3)^2$	6	(3,6)
6	$\frac{2}{3}(6)^2$	24	(6,24)



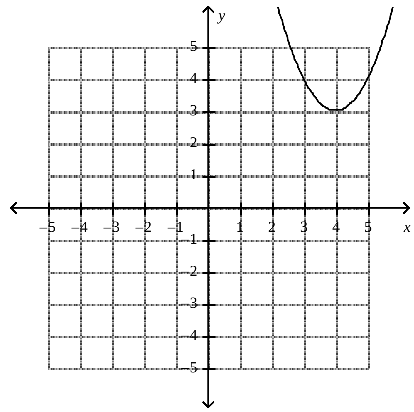
Identify the vertex and axis of symmetry of each graph.

3.



V(-2,-4)
x = -2

4.



V(4,3)
x = 4

V. Without graphing describe how each graph differs from the graph of $y = x^2$.

5. $y = -0.25x^2 + 5$ *Opens down, vertex at (0,5), wider than normal*

6. $y = 3x^2 – 5$ *Opens up, vertex at (0,-5), narrower than normal*