Unit 11 Radical Expressions and Equations

Section 1-- Simplifying Radicals

<u>Radical Expressions</u> – any expression that contains a radical $\sqrt{}$ sign.

$\sqrt{2}$ Radicand – stuff under a radical sign.

<u>Multiplication Property of Square Roots:</u>

For every number zero a and b $\geq$ 0 : $\sqrt{ab} = \sqrt{a} \bullet \sqrt{b}$

Examples: $\sqrt{45} = \sqrt{9} \bullet \sqrt{5} = 3 \bullet \sqrt{5} = 3\sqrt{5}$

Removing Perfect Square Factors:

1) $\sqrt{75} = \sqrt{25} \bullet \sqrt{3} = 5 \bullet \sqrt{3} = 5\sqrt{3}$

2) $4\sqrt{147} = 4 \bullet \sqrt{49} \bullet \sqrt{3} = 4 \bullet 7 \bullet \sqrt{3} = 28\sqrt{3}$

Removing Variable Factors:

1) $\sqrt{48x^5} = \sqrt{16 \bullet 3 \bullet x^2 \bullet x^2 \bullet x} = 4 \bullet \sqrt{3} \bullet x \bullet x \bullet \sqrt{x} = 4x^2\sqrt{3x}$

2) $-b\sqrt{27b^5} = -b \bullet \sqrt{9 \bullet 3 \bullet b^2 \bullet b^2 \bullet b}$

$= \text{-b} \bullet \sqrt{9} \bullet \sqrt{3} \bullet \sqrt{b^2} \bullet \sqrt{b^2} \bullet \sqrt{b}$

$= \text{-b} \bullet 3 \bullet \sqrt{3} \bullet \text{b} \bullet \text{b} \bullet \sqrt{b}$

$= \text{-3b}^3\sqrt{3b}$

3) $\sqrt{8a^7b^6}$

$= \sqrt{4 \bullet 2 \bullet a^2 \bullet a^2 \bullet a^2 \bullet a \bullet b^2 \bullet b^2 \bullet b^2}$

$= \sqrt{4} \bullet \sqrt{2} \bullet \sqrt{a^2} \bullet \sqrt{a^2} \bullet \sqrt{a^2} \bullet \sqrt{a} \bullet \sqrt{b^2} \bullet \sqrt{b^2} \bullet \sqrt{b^2}$

$= 2 \bullet \sqrt{2} \bullet a \bullet a \bullet a \bullet \sqrt{a} \bullet b \bullet b \bullet b$

$= 2a^3b^3\sqrt{2a}$

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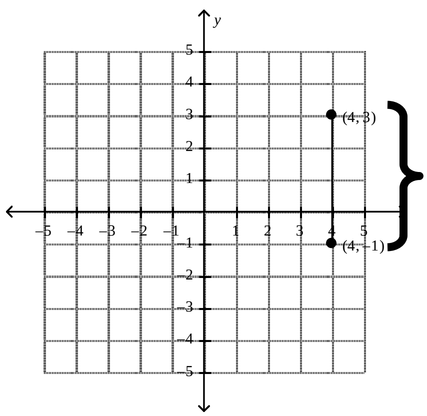
Section 3-- Distance and Midpoint Formula

Finding the distance between two points on a coordinate plane.

When finding the distance on a coordinate plane you can simply count or use Pythagorean Theorem.

Or

The Distance Formula: The distance (d) between two points (x_1,y_1) and (x_2,y_2) is $d = \sqrt{(x_2 - x_1) + (y_2 - y_1)}$



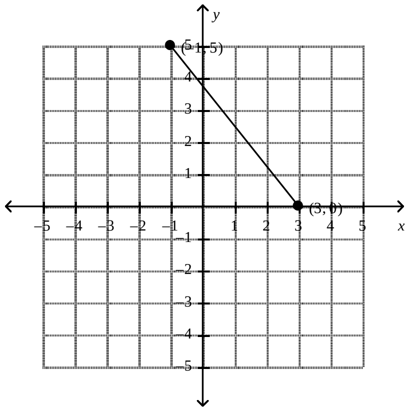
If this is on a graph you can count length (same as distance) is 4 units.

OR

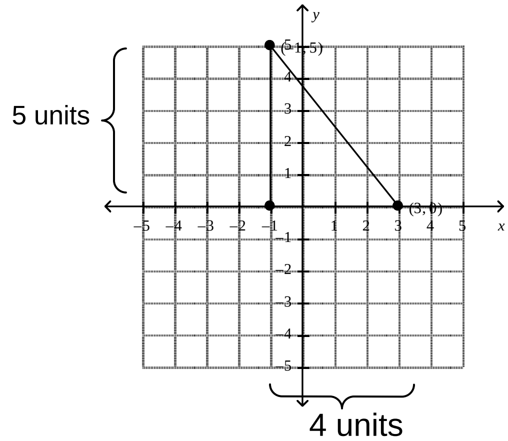
Use Formula:

Write points	(4 , 3) and (4 , -1)
Label points	(x_1,y_1) and (x_2,y_2)
Plug in Formula	$d = \sqrt{(x_2 - x_1) + (y_2 - y_1)}$
Evaluate	$d = \sqrt{(4-4)^2 + ((-1)-3)^2}$
	$d = \sqrt{0^2 + (-4)^2}$
	$d = \sqrt{0 + 16}$
	$d = \sqrt{16}$
Solution	$d = 4$

1. Problem



2. Making a right triangle



3) Solve using Pythagorean Theorem

$a^2 + b^2 = c^2$

$5^2 + 4^2 = c^2$

$25 + 16 = c^2$

$\sqrt{41} = \sqrt{c^2}$

$\sqrt{41} = c$

Or

Use the Distance Formula

The endpoints of the segment above are (-1 , 5) and (3, 0).
(x_1,y_1) and (x_2,y_2)

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(3-(-1))^2 + (0-5)^2}$

$d = \sqrt{4^2 + (-5)^2}$

$d = \sqrt{16 + 25}$

$d = \sqrt{41}$

Finding the Midpoint of a Line Segment

The Midpoint Formula:

The midpoint M of a line segment with endpoints A(x_1,y_1) and B(x_2,y_2) is

$$\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$$

This is basically averaging the x-values and y-values.

1. Either find the coordinates of the two endpoints or you may be given the two points.
2. Average the x –coordinates
3. Average the y – coordinates
4. Write the midpoint in ordered pair format

Examples:

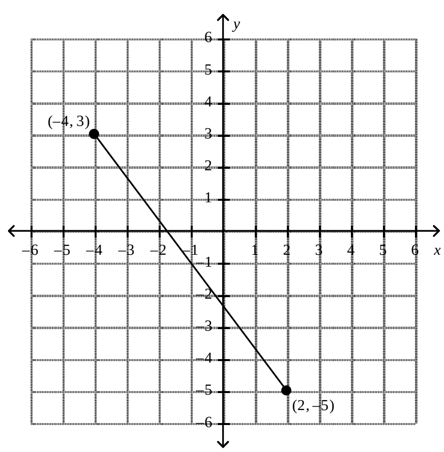
1. (-2, -1), (6,-1) = $\frac{-2+6}{2}, \frac{-1+-1}{2} \;=\; 2, -1$

2. (10,-7), (-2,3) = $\frac{10+(-2)}{2}, \frac{-7+3}{2} \;=\; 4, -2$

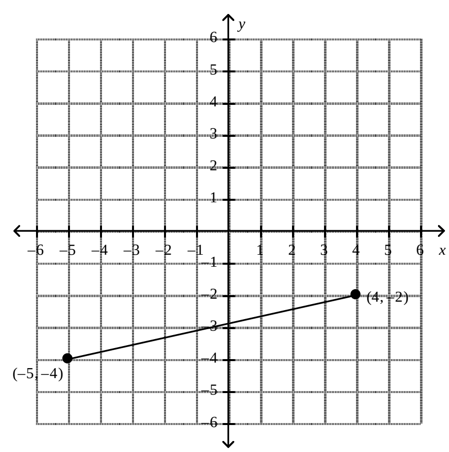
Now You Try!!

Find the distance (length) of the line segment.

1.



2.



For each pair of endpoints, find the distance and midpoint.

3. (5,0) and (12,0)

4. (-6,-4) and (3,-3)

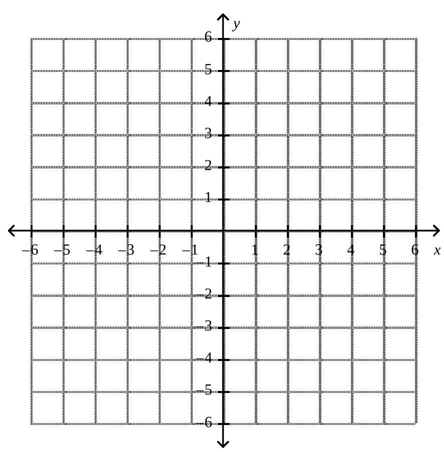
5. (9,-2) and (7,2)

6. (0,6) and (-4,-1)

7. (-8,1) and (1,-2)

8. (1,-2) an (3,4)

9. A triangle CAT has vertices C(-6,5), A(2.-1), and T(-2,-2). Find the perimeter of the triangle. Round to the nearest tenth.



Suggestion – Graph the points, find the length (distance) of each side and then find the perimeter.

Key:

1. 10
2. $\sqrt{85}$
3. 7, (8.5,0)
4. $\sqrt{82}$, (-1.5, -3.5)
5. $2\sqrt{4}$, (8,0)
6. $\sqrt{65}$, (-2,2.5)
7. $3\sqrt{10}$,(-3.5,-0.5)
8. $4\sqrt{10}$, (2,1)
9. CA = 10

   AT ≈ 4.1

   TC ≈ 8.1

   Perimeter ≈ 22.2

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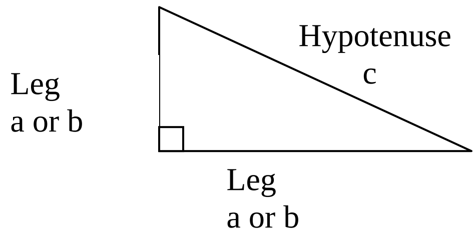
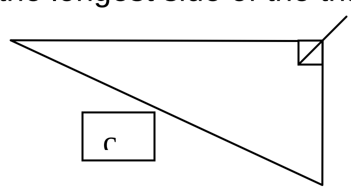
Section 2-- Pythagorean Theorem

Pythagorean Theorem - In any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

$$a^2 + b^2 = c^2$$



Identify the **<u>hypotenuse</u>** first in any problem, draw an line using the right angle as the arrow tip. This is the longest side of the triangle or c in the equation.

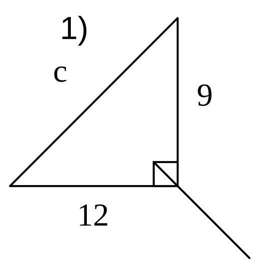


The **<u>legs</u>** are the shorter two sides that form the right angle.

The Pythagorean Theorem is used to solve for a missing sides of right triangles. When faced with a problem that does not have a picture – draw one!!!!

Examples:

1)



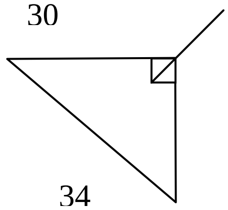
$a^2 + b^2 = c^2$	Identify Parts
$9^{2\,+}\,12^2 = c^2$	Fill in Values
$81 + 144 = c^2$	Exponents
$225 = c^2$	Add
$\sqrt{225} = \sqrt{c^2}$	Square Root each side
$15 = c$	Solution

Check : $a^2 + b^2 = c^2$

$9^{2\,+}\,12^2 = 15^2$

$81 + 144 = 225$

$225=225$

2) 

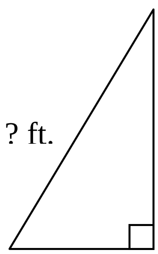
$a^2 + b^2 = c^2$
$30^2 + b^2 = 34^2$
$900 + b^2 = 1156$
$-900 \quad -900$
$b^2 = 256$
$\sqrt{b^2} = \sqrt{256}$
$b = 16$

Check:
$30^2 + 16^2 = 34^2$
$900 + 256 = 1156$
$1156 = 1156$

Problem Solving:

3) A wire from the top of a 24 foot pole is attached to a point 7 feet from the base of the pole. Find the length of the wire?

Draw a picture



We have to assume that the ground (base) meets the pole at 90°. (This assumption is common in many Pythagorean Theorem problems)

Solve for missing hypotenuse

$a^2 + b^2 = c^2$
$7^2 + 24^2 = c^2$
$49 + 576 = c^2$
$625 = c^2$
$\sqrt{625} = \sqrt{c^2}$
$25 = c$

Identifying Right Triangles

The converse of the Pythagorean Theorem

If a triangle has sides of length a, b, and c, and, $a^2 + b^2 = c^2$, the triangle is a right triangle.

What does this mean??

If you know all three sides of a triangle, you can prove that it is a right triangle.

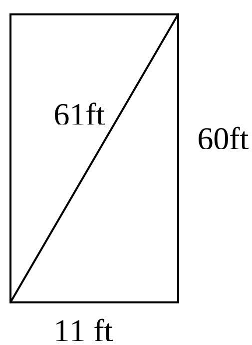
Examples:
Determine if the following measures of a triangle form a right triangle.

1) 5,7,9 $a^2 + b^2 = c^2$ Formula
$5^2 + 7^2 = 9^2$ Fill in – Remember c is the longest side
$25 + 49 = 81$
$74 \neq 81$ The two sides are not equal.
This is not a right triangle.

2) 9m, 40m, 41m $a^2 + b^2 = c^2$
$9^2 + 40^2 = 41^2$
$81 + 1600 = 1681$
$1681 = 1681$
This is a right triangle.

Problem Solving

3) When laying a rectangle driveway a concrete worker measures along one side a distance of 11 feet and along the other side a distance of 60 feet. The measure of the hypotenuse is 61 feet. Are the corners of the rectangle square?



$a^2 + b^2 = c^2$
$11^2 + 60^2 = 61^2$
$121 + 3600 = 3721$
$3721 = 3721$

Yes, the corners meet at right angles.

Now you try!!!

Find the missing side.

	a	b	c
1.	9	12	?
2.	?	20	25
3.	1.5	?	2.5

4. We want to send a fishing rod to Uncle Roy. We are going to send it in a basically flat box. The 2-meter rod will fit in the bottom of the box across the diagonal. The length of the box is 1.6 meters, what is the width of the box.

5. A 10 – foot ladder is placed 6 feet from the base of a building. How high on the building will the ladder reach?

6. Televisions are given by the measurement of their diagonal length. If you have a 25-inch television then the measure of the diagonal is 25 inches. The width of the television is 7 inches, what is the length?

7. Gymnastic floor routines are performed on a 30-foot by 30-foot square mat. The diagonal of the mat is typically used because it gives the gymnast more room. To the nearest whole number, how long is the diagonal?

8. A carpenter braces a wall with a 5 foot board diagonally nailed across the wall. The length of the wall is 13 feet, how wide is the wall??

Determine if the given lengths can be sides of a right triangle.

9. 1in, 1in, 2in

10. 45mm, 60mm, 75mm

11. 2m, 4m, 5m

12. 16ft., 30ft., 34ft.

13. A builder is laying the foundation for a house. One side is 60 feet, the other is 80 feet, and the hypotenuse is 100 feet. Are the corners at right angles.

Key:

1. c = 15
2. a = 15
3. b = 2
4. w = 1.2
5. a = 8
6. length = 24
7. 42 feet
8. 12 feet
9. No
10. Yes
11. No
12. Yes
13. Yes

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Section 4: Operations with Radical Expression

<u>Like Radicals</u> – Radicals with the same radicand when simplified. (Same thing under the radical sign)

Examples: $4\sqrt{7}$ and $-12\sqrt{7}$

$\sqrt{40}$ and $\sqrt{490}$ because it simplifies to $2\sqrt{10}$ and $7\sqrt{10}$

<u>Unlike Radicals</u> – Radicals with the same radicand when simplified. (Different thing under the radical sign)

Examples: $3\sqrt{11}$ and $2\sqrt{5}$

$\sqrt{90}$ and $\sqrt{400}$ because it simplifies to $3\sqrt{10}$ and 20

Simplify each expression:

Add the numbers in front of the radical and write as one radical.

1. $8\sqrt{6}+3\sqrt{6}=(8+3)\sqrt{6}=11\sqrt{6}$

2. $5\sqrt{12}-9\sqrt{12}=(5-9)\sqrt{12}=-4\sqrt{12}$

3. $4\sqrt{3}+3\sqrt{3}-6\sqrt{3}=(4+3-6)\sqrt{3}=1\sqrt{3}=\sqrt{3}$

4.
$2\sqrt{48}-3\sqrt{75}$ — Simplify radicals

$2\sqrt{16\bullet 3}-3\sqrt{25\bullet 3}$

$2\bullet 4\sqrt{3}-3\bullet 5\sqrt{3}$

$8\sqrt{3}-15\sqrt{3}$ — Combine like radical

$-7\sqrt{3}$

5.
$-3\sqrt{5}+9\sqrt{2}+5\sqrt{2}+5\sqrt{5}$

$-3\sqrt{5}+5\sqrt{5}+9\sqrt{2}+5\sqrt{2}$ — Use Commutative Property

$(-3+5)\sqrt{5}+(9+5)\sqrt{2}$ — Combine like radicals

$2\sqrt{5}+14\sqrt{2}$

Use the Distributive Property

Examples:

1. $\sqrt{5}\,(\sqrt{7} - 8)$

$(\sqrt{5} \cdot \sqrt{7}) - (\sqrt{5} \cdot 8)$

Multiply each addend by factor

$\sqrt{35} - 8\sqrt{}$

Solution / Simplify if possible

2. $\sqrt{2a}\ (\sqrt{3} + \sqrt{ba})$

$(\sqrt{2a}\ \sqrt{3}) + (\sqrt{2a} \cdot \sqrt{ba})$

$\sqrt{6a} + \sqrt{12a^2}$

$\sqrt{6a} + \sqrt{4\,a^2 3}$

$\sqrt{6a} + 2a\sqrt{3}$

Using FOIL (First Outer Inner Last)

Reminder $(a+b)(c+d) = F + O + I + L$

$ac + ad + bc + bd$

3. $(2\sqrt{5} + 3\sqrt{7})\ (\sqrt{5} - 5\sqrt{7})$

$(2\sqrt{5} \cdot \sqrt{5}) + (2\sqrt{5} \cdot -5\sqrt{7}) + (3\sqrt{7} \cdot \sqrt{5}) + (3\sqrt{7} \cdot -5\sqrt{7})$

$2\sqrt{25} + -10\sqrt{35} + 3\sqrt{35} + -15\sqrt{49}$

$2 \cdot 5 - 10\sqrt{35} + 3\sqrt{35} - 15(7)$

$10 - 7\sqrt{35} - 105$

$-95 - 7\sqrt{35}$

4. $(\sqrt{10} + 3)^2$

$(\sqrt{10} + 3)\ (\sqrt{10} + 3)$

$(\sqrt{10} \cdot \sqrt{10}) + (\sqrt{10} \cdot 3) + (3 \cdot \sqrt{10}) + (3)\ (3)$

$\sqrt{100} + 3\sqrt{10} + 3\sqrt{10} + 9$

$10 + 6\sqrt{10} + 9$

$19 + 6\sqrt{10}$

***Rationalizing Denominators with Conjugates**:*

Conjugates – The sum and difference of the same two terms.

Examples of conjugates:

1) $(\sqrt{2} + \sqrt{7})(\sqrt{2} - \sqrt{7})$ 2) $(6 + \sqrt{5})\ 6 - \sqrt{5})$

Rationalizing is the method used to take radicals out of the denominator.
Simplify:

3)

$$\frac{3}{\sqrt{2} + \sqrt{7}} \cdot \frac{(\sqrt{2} - \sqrt{7})}{(\sqrt{2} - \sqrt{7})}$$

Multiply both numerator and denominator by the conjugate of the denominator.

$$\frac{3(\sqrt{2} - \sqrt{7})}{(\sqrt{2} + \sqrt{7})(\sqrt{2} - \sqrt{7})}$$

Use the distributive property
Use FOIL

$$\frac{3\sqrt{2} - 3\sqrt{7}}{\sqrt{2} \cdot \sqrt{2} - \sqrt{2} \cdot \sqrt{7} + \sqrt{7} \cdot \sqrt{2} - \sqrt{7} \cdot \sqrt{7}}$$

Collect Like Terms

$$\frac{3\sqrt{2} - 3\sqrt{7}}{\sqrt{4} - \sqrt{14} + \sqrt{14} - \sqrt{49}}$$

Simplify

$$\frac{3\sqrt{2} - 3\sqrt{7}}{2 - 7}$$

Solution

$$\frac{3\sqrt{2} - 3\sqrt{7}}{-5}$$

Solution

4) $$\frac{4}{(5 + \sqrt{6})} \cdot \frac{(5 - \sqrt{6})}{(5 - \sqrt{6})}$$

$$\frac{4\ (5 - \sqrt{6})}{(5 + \sqrt{6})(5 - \sqrt{6})}$$

$$\frac{4 \cdot 5 - 4\sqrt{6}}{5 \cdot 5 - 5 \cdot \sqrt{6} + \sqrt{6} \cdot 5 - \sqrt{6} \cdot \sqrt{6}}$$

$$\frac{20 - 4\sqrt{6}}{25 - \sqrt{36}}$$

$$\frac{20 - 4\sqrt{6}}{25 - 6}$$

$$\frac{20 - 4\sqrt{6}}{19}$$

Now you try!!!

Simplify each expression.

1. $-6\sqrt{7}+4\sqrt{7}$

2. $13\sqrt{10}-5\sqrt{10}$

3. $5\sqrt{63}-\sqrt{28}$

4. $\sqrt{12}+2\sqrt{27}$

5. $5\sqrt{3}+4\sqrt{3}-7\sqrt{3}$

6. $2\sqrt{20}-\sqrt{80}-\sqrt{45}$

7. $\sqrt{3}\ \ 3+2\sqrt{2}$

8. $\sqrt{6}\ \ \sqrt{18}-2$

9. $2\sqrt{5}\ \ 1-\sqrt{5}$

10. $2\sqrt{10}+\sqrt{3}\quad\sqrt{5}-\sqrt{3}$

11. $4-\sqrt{13}$

12. $3\sqrt{2}+\sqrt{3}\quad\sqrt{2}-5\sqrt{3}$

13. $\dfrac{-12}{\sqrt{7}+\sqrt{3}}$

14. $\dfrac{3}{\sqrt{11}-\sqrt{3}}$

15. $\dfrac{24}{\sqrt{6}-\sqrt{18}}$

Key:

1. $-2\sqrt{7}$
2. $8\sqrt{10}$
3. $15\sqrt{7}-2\sqrt{7}$
4. $8\sqrt{3}$
5. $2\sqrt{3}$
6. $3\sqrt{5}$
7. $3\sqrt{3}+3\sqrt{6}$
8. $6\sqrt{3}-2\sqrt{6}$
9. $2\sqrt{5}-10$
10. $10\sqrt{2}-2\sqrt{30}+\sqrt{15}-3$
11. $29-8\sqrt{3}$
12. $-9-14\sqrt{6}$
13. $\frac{-12\sqrt{7}+12\sqrt{3}}{4}$
14. $\frac{3\sqrt{11}+3\sqrt{3}}{8}$
15. $-2\sqrt{6}-6\sqrt{2}$

Multiplying two Radicals

1) $\sqrt{8} \bullet \sqrt{6} = \sqrt{48} = \sqrt{16 \bullet 3} = 4\sqrt{3}$

2) $2\sqrt{3a^3} \bullet 5\sqrt{6a^2b}$

$= 10\sqrt{18a^5b}$

$= 10\sqrt{9} \bullet \sqrt{2} \bullet \sqrt{a^4} \bullet \sqrt{ab}$

$= 10 \bullet 3 \bullet \sqrt{2} \bullet a^2 \bullet \sqrt{ab}$

$= 30a^2\sqrt{2ab}$

Simplifying Radical Expressions Involving Quotients

Division Property of Square Roots

For every number a≥0 and b > 0. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Example: $\sqrt{\frac{12}{25}} = \frac{\sqrt{12}}{\sqrt{25}} = \frac{2\sqrt{3}}{5}$

Simplifying Fractions Within Radicals

1) $\sqrt{\frac{20a^3}{49}} = \frac{\sqrt{20a^3}}{\sqrt{49}} = \frac{2a^2\sqrt{5a}}{7}$

2) $\sqrt{\frac{81}{b^2}} = \frac{\sqrt{81}}{\sqrt{b^2}} = \frac{9}{b}$

Simplifying Radicals by Dividing

1) $\sqrt{\frac{80}{2}} = \sqrt{40} = 2\sqrt{10}$

2) $\sqrt{\frac{16b^5}{4b}} = \sqrt{4b^4} = 2b^2$

Rationalizing a Denominator

Rationalizing a denominator means that we are going to make sure there is no radical left in the denominator. (Like reducing a fraction)

1) $\frac{3}{\sqrt{5}} \bullet \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{\sqrt{25}} = \frac{3\sqrt{5}}{5}$

Since the square root of 5 is in the denominator, If you multiply the denominator and numerator by the square root of 5, then you end up with a perfect square in the denominator.

2) $\sqrt{\frac{5}{7b}} = \frac{\sqrt{5}}{\sqrt{7b}} \bullet \frac{\sqrt{7b}}{\sqrt{7b}} = \frac{\sqrt{35b}}{7b}$

3) $\sqrt{\frac{6m}{2}} = \frac{\sqrt{6m}}{\sqrt{2}} \bullet \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{12m}}{\sqrt{4}} = \frac{2\sqrt{3m}}{2}$

How are you sure a radical is in simplest form:

1. There are no radicals in the denominator.
2. No fractions under the radical sign.
3. There are no perfect square factors under the radical except for 1.

Now you try:

Simplify the radical expressions.

1) $\sqrt{98}$

2) $-5\sqrt{28n^2}$

3) $4\sqrt{108b^5}$

4) $\sqrt{11} \bullet \sqrt{33}$

5) $5\sqrt{15} \bullet \sqrt{3}$

6) $3\sqrt{7} \bullet 5\sqrt{21}$

7) $3\sqrt{2n} \bullet \sqrt{8n}$

8) $\sqrt{8t} \bullet \sqrt{3t^4}$

9) $-2\sqrt{13t^3} \bullet 4t\sqrt{26}$

10) $\sqrt{6a} \bullet a\sqrt{12}$

11) $\dfrac{\sqrt{75}}{\sqrt{3}}$

12) $\dfrac{\sqrt{40}}{\sqrt{10}}$

13) $\dfrac{\sqrt{90a^3}}{\sqrt{2a}}$

14) $\dfrac{\sqrt{32b^5}}{\sqrt{4b^2}}$

15) $\sqrt{\dfrac{5b}{4a^2}}$

16) $\sqrt{\dfrac{27a^3}{64b^2}}$

17) $\sqrt{\dfrac{3}{5x}}$

18) $\sqrt{\dfrac{18}{9b}}$

19) $\sqrt{\dfrac{140x^5}{5x^2}}$

20) $\sqrt{\dfrac{625b}{100ab^3}}$

Answers:

1. $7\sqrt{2}$
2. $-10n\sqrt{7}$
3. $24b^2\sqrt{3b}$
4. $11\sqrt{3}$
5. $15\sqrt{5}$
6. $105\sqrt{3}$
7. 12n
8. $2t^2\sqrt{6t}$
9. $-104t^2\sqrt{2t}$
10. $6a\sqrt{2a}$
11. 5
12. 2
13. $3a\sqrt{5}$
14. $2b\sqrt{2b}$
15. $\frac{\sqrt{5b}}{2a}$
16. $\frac{3a\sqrt{3a}}{8b}$
17. $\frac{\sqrt{15x}}{5x}$
18. $\frac{\sqrt{2b}}{b}$
19. $2x\sqrt{7x}$
20. $\frac{25}{10b}$