

Unit 12 Rational Expressions and Equations

Section 1: Simplifying Rational Expressions

A **rational expression** is a fraction with a variable in the denominator.

Simplifying

$$1. \quad \frac{-8abc}{16ac} = \frac{-8}{16} \cdot \frac{a}{a} \cdot \frac{b}{1} \cdot \frac{c}{c} \quad \text{Separate into like terms / Simplify}$$

$$= \frac{-1}{8} \cdot \frac{b}{1} = \boxed{\frac{-b}{8}} \quad \text{Multiply to find solution}$$

$$2. \quad \frac{-12y^3z}{18y^2z^4} = \frac{-12}{18} \cdot \frac{y^3}{y^2} \cdot \frac{z^3}{z^4}$$

$$= \frac{-2}{3} \cdot \frac{y}{1} \cdot \frac{1}{z}$$

$$= \boxed{\frac{-2y}{3z}}$$

$$3. \quad \frac{3y-9}{4y-12} = \frac{3(y-3)}{4(y-3)} = \frac{3}{4} \quad \text{Factor, Simplify, Solution}$$

$$4. \quad \frac{x^2+3x}{x^2+6x+9} = \frac{x(x+3)}{(x+3)(x+3)} = \frac{x}{x+3}$$

$$5. \quad \frac{a^2-a-6}{a^2+5a+6} = \frac{(a-3)(a+2)}{(a+2)(a+3)} = \frac{a-3}{a+3}$$

Opposite Factors:

$$6. \quad \frac{3x-27}{81-x^2} = \frac{3(x-9)}{(9-x)(9+x)} \quad \text{Factor}$$

$$= \frac{3(x-9)}{-1(x-9)(9+x)} \quad \text{Simplify}$$

$$= \frac{3}{-1(9+x)} \quad \text{Solution}$$

$$= -\frac{3}{9+x}$$

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Section 4: Combining Rational Expressions

I. Like Denominators

<p>1. $\frac{2}{a} + \frac{4}{a}$ same denominators</p> <p>$\frac{2+4}{a}$ add numerators keep denominator</p> <p>$\frac{6}{a}$ simplify if possible</p>	<p>2. $\frac{8}{9y} - \frac{5}{9y}$</p> <p>$\frac{8-5}{9y}$</p> <p>$\frac{3 \div 3}{9y \div 3} = \frac{1}{3y}$</p>
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3. $\frac{b}{b+4} + \frac{b+8}{b+4}$ Add – Keep –Simplify

$$\frac{b+(b+8)}{b+4}$$

$$\frac{2b+8}{b+4}$$

$$\frac{2(b+4)}{(b+4)} = 2$$

II. Unlike Denominators

Remember : Finding LCM (Least Common Multiple) or LCD(Least Common Denominator)

1. Find LCD of 15 and 18 3 $\left| \begin{array}{cc} 15 & 18 \\ \hline 5 & 6 \end{array} \right.$ LCD = 3•5•6=90

2. Find LCD of 12a and 8ab 4 $\left| \begin{array}{cc} 12a & 8ab \\ \hline 3a & 2ab \\ \hline 3 & 2b \end{array} \right.$

LCD = 4•a•3•2b=24ab

How to Add / Subtract with Unlike Denominators:

1. Find the LCD for each problem
2. Rewrite fractions with LCD
3. Add or Subtract
4. Simplify

Examples:

$$\begin{array}{l}
 1. \quad \frac{3}{5m} + \frac{4}{2m^2} \qquad \text{LCD} \quad m \mid \frac{5m}{5} \frac{2m^2}{2m} = 10m^2 \\
 \\
 \frac{3 \cdot 2m}{5m \cdot 2m} + \frac{4 \cdot 5}{2m^2 \cdot 5} \\
 \frac{6m}{10m^2} + \frac{20}{10m^2} \\
 \frac{2(3m+10) \div 2}{10m^2 \div 2} \\
 \frac{3m+10}{5m^2}
 \end{array}$$

$$\begin{array}{l}
 2. \quad \frac{m}{m^2-9} - \frac{3}{m-3} \\
 \\
 \frac{m}{(m+3)(m-3)} - \frac{3}{(m-3)} \\
 \\
 \frac{m \cdot 1}{(m+3)(m-3) \cdot 1} - \frac{3 \cdot (m+3)}{(m-3)(m+3)} \\
 \\
 \frac{m-3m-9}{(m+3)(m-3)} \\
 \\
 \frac{-2m-9}{(m-3)(m+3)}
 \end{array}
 \qquad
 \begin{array}{l}
 3. \quad \frac{5b}{2b+6} - \frac{b}{b+3} \\
 \\
 \frac{5b \cdot 1}{2(b+3) \cdot 1} - \frac{b \cdot 2}{(b+3) \cdot 2} \\
 \\
 \frac{5b-2b}{2(b+3)} \\
 \\
 \frac{3b}{2(b+3)}
 \end{array}$$

Now You Try!!!!

1. $\frac{7}{m} + \frac{4}{m}$

2. $\frac{4t}{2} - \frac{5t}{2}$

3. $\frac{8}{y-2} - \frac{6}{y-2}$

4. $\frac{2x}{x+3} + \frac{6}{x+3}$

5. $\frac{8m+3}{3m+4} - \frac{2m-5}{3m+4}$

6. $\frac{3}{4m^3} + \frac{5}{8m^2}$

7. $\frac{3}{x+6} - \frac{4x}{x^2-36}$

8. $\frac{t}{5} - \frac{t}{15}$

9. $\frac{6w}{7a} + \frac{2w}{a^3}$

10. $\frac{2}{x+1} + \frac{x^2+4x-5}{x^2-2x-3}$

11. $\frac{4a}{2a+6} + \frac{3}{a+3}$

12. $\frac{10k}{3k+1} - \frac{k}{3+9k}$

Key:

1. $\frac{11}{m}$

2. $\frac{-t}{2}$

3. $\frac{2}{y-2}$

4. 2

5. 2

6. $\frac{5m+6}{8m^3}$

7. $\frac{-x-18}{x^2-36}$

8. $\frac{2t}{15}$

9. $\frac{6a^2w+14w}{7a^3}$

10. $\frac{x^2+6x-11}{(x+1)(x+3)}$

11. $\frac{2a+3}{a+3}$

12. $\frac{29k}{3(3k+1)}$

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Section 3: Dividing Polynomials

*Dividing a polynomial by a monomial:
(many terms) (one term)*

1. $12x^3 + 4x^2 + 16x \div 4x$

$$\frac{12x^3 + 4x^2 + 16x}{4x}$$

Rewrite

$$\frac{12x^3}{4x} + \frac{4x^2}{4x} + \frac{16x}{4x}$$

Separate into parts

$$3x^2 + x + 4$$

Solution

2. Remember Long Division

$$\begin{array}{r} \text{quotient} \\ \text{divisor} \overline{) \text{dividend}} \end{array}$$

A look back: $473 \div 18$

$$\begin{array}{r} 26R5 \\ 18 \overline{) 473} \\ \underline{-36} \\ 113 \\ \underline{-108} \\ 5 \end{array}$$

Is

$$26\frac{5}{18}$$

We will be using long division to divide polynomials.

1. $(x^2 + 4x + 3) \div (x + 3)$

$$\begin{array}{r} \text{dividend} \quad \text{divisor} \\ \begin{array}{r} x+1 \\ x+3 \overline{) x^2 + 4x + 3} \\ \underline{-(x^2 + 3x)} \\ x+3 \\ \underline{-(x+3)} \\ 0 \end{array} \end{array}$$

Set up like long division

Match the first term ($x \cdot x = x^2$), Multiply $x(x+3)$

Subtract and bring down rest of dividend

Multiply $x+3(1)$, Subtract to find Remainder

The Solution is $(x+1)$.

$$(x^2 - 72 + 10) \div (x - 2)$$

$$2. \quad \begin{array}{r} x-5 \\ x-2 \overline{)x^2 - 72 + 10} \\ \underline{-(x^2 + 2x)} \\ -5x + 10 \\ \underline{-(-5x + 10)} \\ 0 \end{array}$$

$$(x^2 - 72 + 10) \div (x - 2) = x - 5$$

$$(2a^2 + 7a + 3) \div (a + 2)$$

$$3. \quad \begin{array}{r} 2a+3 \\ a+2 \overline{)2a^2 + 7a + 3} \\ \underline{-(2a^2 + 4a)} \\ 3a + 3 \\ \underline{-(3a + 6)} \\ -3 \end{array}$$

$$(2a^2 + 7a + 3) \div (a + 2) = 2a + 3 - \frac{3}{a + 2}$$

A remainder should be placed over the divisor for the answer.

Whenever a term is skipped a zero term **must** be added!!

Example: $4x^2 + 16$ should be $4x^2 + 0x + 16$
 $3x^3 + 2x$ should be $3x^3 + 0x^2 + 2x + 0$

$$(a^3 + 8a - 20) \div (a - 2)$$

$$4. \quad \begin{array}{r} a^2 + 2a + 12R4 \\ a-2 \overline{)a^3 + 0a^2 + 8a - 20} \\ \underline{-(a^3 - 2a^2)} \\ 2a^2 + 8a - 20 \\ \underline{-(2a^2 - 4a)} \\ 12a - 20 \\ \underline{-(12a + 24)} \\ 4 \end{array}$$

$$(a^3 + 8a - 20) \div (a - 2) = a^2 + 2a + 12 + \frac{4}{a - 2}$$

Now you try!!

1. $(12y - 4) \div (3y - 1)$

2. $(x^2 - 3x) \div (x - 3)$

3. $(s^2 + 11s + 18) \div (s + 2)$

4. $(6r^3 - 15r^2) \div (2r - 5)$

5. $(2m^2 + 7m + 3) \div (m + 2)$

6. $(3x^2 - 10x - 24) \div (3x - 4)$

7. $(a^3 + 27) \div (a + 3)$

8. $(y^2 - 5y) \div (y - 5)$

9. $(2x^2 + 3x - 6) \div (2x - 1)$

10. $(a^3 + 8a - 21) \div (a - 2)$

Key:

1. 4

2. x

3. s+9

4. $3r^2$

5. $2m + 3 - \frac{3}{m + 2}$

6. $x - 2 - \frac{16}{3x - 4}$

7. $a^2 + 2a + 12 + \frac{3}{a - 2}$

8. y

9. $x + 2 - \frac{4}{2x - 1}$

10. $a^2 + 2a + 12 + \frac{3}{a - 2}$

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Section 2: Multiplying and Dividing Rational Expressions

Review Multiplying and Dividing Fractions:

Multiply:

Method 1: $\frac{3}{7} \cdot \frac{1}{3} = \frac{\cancel{3}^1}{7} \cdot \frac{1}{\cancel{3}_1} = \frac{1}{7}$ Simplify then multiply.

Method 2: $\frac{3}{7} \cdot \frac{1}{3} = \frac{3 \div 3}{21 \div 3} = \frac{1}{7}$ Multiply, Simplify(Reduce)

Divide:

$$\frac{3}{5} \div \frac{1}{5} = \frac{3}{5} \cdot \frac{5}{1} = \frac{15 \div 5}{5 \div 5} = \frac{3}{1} = 3$$

Or

$$\frac{3}{5} \div \frac{1}{5} = \frac{3}{5} \cdot \frac{5}{1} = \frac{3}{1} = 3$$

Multiply Rational Expressions:

$$1. \quad \frac{12x}{5y} \cdot \frac{20y^2}{36x^2} = \frac{\cancel{12}^1 \cancel{x}}{\cancel{5}_1 \cancel{y}} \cdot \frac{\cancel{20}^4 \cancel{y} \cdot y}{\cancel{36}_3 \cdot x \cdot \cancel{x}} = \frac{4y}{3x}$$

$$2. \quad \frac{\cancel{(x-2)}^2}{\cancel{2}_2} \cdot \frac{\cancel{3}^3 \cancel{12}^1 x}{(x+2)\cancel{(x-2)}} = \frac{3x}{2(x+2)}$$

$$3. \quad \frac{b^2 + 6b + 9}{b^2 - 9} \cdot \frac{b-2}{b+3}$$

$$\frac{\cancel{(b+3)}^1 \cancel{(b+3)}^1 + 3 \textcircled{1}}{\cancel{(b-3)}^1 \cancel{(b+3)}^1 + 2 \textcircled{1}} \cdot \frac{b-2}{b+3}$$

$$\frac{b-2}{b+3}$$

Dividing Rational Expressions:

$$4. \quad \frac{6x^3}{y} \div \frac{2x}{y^2} = \frac{6x^3}{y} \cdot \frac{y^2}{2x} = \frac{6}{2} \cdot \frac{x^3}{x} \cdot \frac{y^2}{y} = \frac{3x^2y}{1} = 3x^2y$$

$$5. \quad \frac{2m+8}{m+5} \div m+4$$

$$\frac{2m+8}{m+5} \div \frac{m+4}{1}$$

Put m+4 over 1

$$\frac{2m+8}{m+5} \cdot \frac{1}{m+4}$$

Change division to multiplication

$$\frac{\cancel{2(m+4)}}{(m+5)\cancel{(m+4)}}$$

Put together / simplify

$$\frac{2}{m+5}$$

Solution

$$6. \quad \frac{x^2-9}{8x^3} \div \frac{6-2x}{4x}$$

$$\frac{(x+3)(x-3)}{8x^3} \cdot \frac{4x}{2(3-x)}$$

Flip and Factor

$$\frac{(x+3)(x-3)}{8x^3} \cdot \frac{4x}{-2(x-3)}$$

Pull -1 out

$$\frac{-4x(x+3)\cancel{(x-3)}}{-16x^3\cancel{(x-3)}}$$

Simplify

$$-\frac{(x+3)}{4x^2}$$

Now You Try!!!!

Multiply:

$$1. \quad \frac{4a^2b}{6b^2c} \cdot \frac{3ab}{2c}$$

$$2. \quad \frac{7n}{n-2} \cdot \frac{3(n-2)}{28}$$

$$3. \quad \frac{4y+8}{y^2-2y} \cdot \frac{y-2}{y+2}$$

$$4. \quad \frac{a+5}{3a+6} \cdot \frac{3a^2+6a}{a^2+2a-15}$$

$$5. \quad \frac{x^2+9x+18}{2x+1} \cdot \frac{6x+3}{x+6}$$

Divide

$$6. \quad \frac{a^2+13a+40}{a-7} \div \frac{a+8}{a^2-49}$$

$$7. \quad \frac{3w+12}{5w} \div \frac{w+4}{10w}$$

$$8. \quad \frac{x-4}{8} \div \frac{4-x}{3}$$

$$9. \quad \frac{x+3}{x+1} \div x^2+5x+6$$

$$10. \quad \frac{x^2+7x+10}{x-1} \div \frac{x^2+2x-15}{x-1}$$

Key:

1. $\frac{a^3}{c^2}$

2. $\frac{3n}{4}$

3. $\frac{4}{y}$

4. $\frac{a}{a-3}$

5. $3(x+3)$

6. $(x+5)(x+7)$

7. 6

8. $-\frac{3}{8}$

9. $\frac{1}{(x+1)(x+2)}$

10. $\frac{x+2}{x-3}$

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Section 5: Solving Rational Equations

Rational Equation – An equation that contains a rational expression (fraction)

Examples:

$$\frac{2}{3x} - \frac{1}{2x} = \frac{1}{6}$$

1. $\frac{2 \cdot 2}{3x \cdot 2} - \frac{1 \cdot 3}{2x \cdot 3} = \frac{1}{6}$ Find LCD

$$\frac{4-3}{6x} = \frac{1}{6}$$

Combine

$$\frac{1}{6x} = \frac{1}{6}$$

Cross Multiply

$$6 = 6x$$

$$\frac{6}{6} = \frac{6x}{6}$$

Solve Algebraic

1 = x

Check Solution:

$$\frac{2}{3(1)} - \frac{1}{2(1)}$$

$$\frac{2}{3} - \frac{1}{2}$$

$$\frac{4-3}{6}$$

$\frac{1}{6}$

$$\frac{x}{7} = \frac{x+3}{10}$$

2. $10x = 7(x+3)$ Cross Multiply

$$10x = 7x + 21$$

Solve Algebraic Equation

$$\frac{-7x}{-7x} \quad \frac{-7x}{-7x}$$

$$\frac{3x}{3} = \frac{21}{3}$$

x = 7

Check Solution:

$$\frac{7}{7} = \frac{7+3}{10}$$

$$1 = \frac{10}{10}$$

1 = 1

$$\frac{6}{x} - \frac{3}{2x} = \frac{1}{2}$$

$$\frac{6 \cdot 2}{x \cdot 2} - \frac{3}{2x} = \frac{1}{2}$$

$$3. \quad \frac{12-3}{2x} = \frac{1}{2}$$

$$\frac{18}{2} = \frac{2x}{2}$$

$9 = x$

Check solution:

$$\frac{6}{9} - \frac{3}{2(9)}$$

$$\frac{12-3}{18}$$

$$\frac{9}{18}$$

$\frac{1}{2}$

$$\frac{3}{x} + \frac{4x}{x-3} = 4$$

$$\frac{3(x-3)}{x(x-3)} + \frac{4x \cdot x}{(x-3)x} = 4$$

$$4. \quad \frac{3x-9+4x^2}{x(x-3)} = 4$$

$$\frac{4x^2+3x-9}{x(x-3)} = \frac{4}{1}$$

$$4x^2+3x-9 = 4x(x-3)$$

$$4x^2+3x-9 = 4x^2-12x$$

$$\begin{array}{r} -4x^2 \\ \underline{-4x^2} \\ 3x-9 = -12x \end{array}$$

$$\begin{array}{r} -3x \\ \underline{-3x} \\ -9 = -15x \end{array}$$

$$\frac{-9}{-15} = \frac{-15x}{-15}$$

$\frac{3}{5} = x$

Check Solution:

$$\frac{3}{\frac{3}{5}} + \frac{4(\frac{3}{5})}{\frac{3}{5}-3}$$

$$5 + \frac{(\frac{12}{5})}{(-\frac{12}{5})}$$

$$5 + (-1)$$

4

Now You Try!!!

1. $\frac{a}{6} + \frac{2a}{3} = \frac{-5}{2}$

2. $\frac{t-1}{4} = \frac{t}{3}$

3. $\frac{2a-3}{6} = \frac{2a}{3} + \frac{1}{2}$

4. $\frac{4}{3y} + \frac{1}{y} = 7$

5. $\frac{5}{5-p} - \frac{1}{5-p} = -2$

6. $\frac{5}{2x} - \frac{1}{6x} = 2$

7. $\frac{1}{4x} + \frac{1}{6x} = 5$

8. $\frac{n-3}{n} = \frac{n-3}{n-6}$

9. $\frac{3}{r+4} - \frac{1}{r} = \frac{1}{r}$

10. $\frac{6}{t+1} - \frac{3}{4t+4} = \frac{3}{4}$

Key:

1. -3

2. -3

3. -3

4. $\frac{1}{3}$

5. 7

6. $\frac{7}{6}$

7. $\frac{1}{12}$

8. 3

9. 8

10.6

Now you try!!!!

1. $\frac{25ab}{30b^2}$

2. $\frac{-8y^4z}{20y^6z^2}$

3. $\frac{5(x-1)}{8(x-1)}$

4. $\frac{x^2-4x}{3(x-4)}$

5. $\frac{x^2-6x}{x^2-4x-12}$

6. $\frac{v^2+6v+9}{v^2-2v-15}$

7. $\frac{y^2+7y+10}{y^2+5y}$

8. $\frac{x-6}{36-x^2}$

9. $\frac{6x^2-x-12}{8x^2-10x-3}$

10. $\frac{6-2x}{x-3}$

Key:

1. $\frac{5a}{6b}$

2. $\frac{-2}{5y^2z}$

3. $\frac{5}{8}$

4. $\frac{x}{3}$

5. $\frac{x}{x+2}$

6. $\frac{v+3}{v-5}$

7. $\frac{y+2}{y}$

8. $\frac{-1}{x+8}$

9. $\frac{3x+4}{4x+1}$

10. -2