

Unit 2 – Solving Equations

Section 1 – Solving One-Step Equations

(Refer to Pages 89 - 90, problem numbers 1 – 42 and Page 96, problem numbers 1 – 44 in your textbook for additional practice)

Vocabulary and Examples:

Inverse Operations – when solving a one-step equation, isolate the variable by moving numbers or letters from one side of the equation to the other. This is done by doing the opposite (or inverse) of the operation that is happening to the number or letter that is to be moved.

REMEMBER: WHAT IS DONE TO ONE SIDE OF THE EQUATION **MUST** BE DONE TO THE OTHER SIDE OF THE EQUATION.

Solving One-Step Equations – to solve a one-step equation (an equation that requires one step to isolate the variable):

First, move any number that is on the same side of the equal sign as the letter to the other side of the equal sign. To do this, do the inverse operation. This step will isolate the variable on one side of the equation and move all of the numbers to the other side of the equation.

Once the numbers are moved to the same side of the equation, combine the numbers together by paying attention to the signs attached to the numbers.

Ex 1: $x - 3 = -8$ isolate the “x” by moving the -3 from where it is to the other side of the equation so that it can be combined with the -8, since -3 and -8 are like terms.

To move the “-3” do the inverse (opposite) of “minus 3”, which is “add 3” to both sides of the equation.

$$\begin{array}{r} x - 3 = -8 \\ +3 = +3 \\ \hline \end{array}$$

$x = -5$ check your answer by replacing “x” with your answer to see if it is a true statement

CHECK: $-5 - 3 = -8$
 $-8 = -8$ Because this is a true statement the answer is correct

Ex 2: $15 = a + 4.2$ To isolate the “a” move the “+4.2” to the other side of the equation so that it can be combined with 15, since 15 and “+4.2” are like terms.

To move the “+4.2” do the inverse (opposite) of “plus 4.2”, which is “subtract 4.2” to both sides of the equation.

$$\begin{array}{r} 15 = a + 4.2 \\ -4.2 = -4.2 \\ \hline \end{array}$$

$10.8 = a$ check your answer by replacing “a” with your answer to see if it is a true statement

CHECK: $15 = 10.8 + 4.2$
 $15 = 15$ Because this is a true statement the answer is correct

Ex 3: $-\frac{r}{4} = -10.4$

NOTICE: there is a negative sign that is outside of the fraction. Put it EITHER in the numerator

or denominator so that it either becomes $\frac{-r}{4}$ or $\frac{r}{-4}$

It is ALWAYS better to assign the negative sign to the number and not the letter.

So the problem to work with is:

$$\frac{r}{-4} = -10.4$$

To isolate the "r" the "-4" needs to move from where it is to the other side of the equation so that it can be combined with -10.4, since -4 and -10.4 are like terms.

To move the "-4" do the inverse (opposite) of "divide by -4", which is "multiply by -4" to both sides of the equation.

REMEMBER: IT IS THE OPPOSITE OF THE OPERATION THAT IS BEING DONE, NOT THE OPPOSITE OF THE SIGN!!

$$\frac{r}{-4} = -10.4$$

$$\cancel{-4} \times -4 \quad \cancel{x -4}$$

r = 41.6 check your answer by replacing "r" with your answer to see if it is a true statement

CHECK: $\frac{41.6}{-4} = -10.4$

-10.4 = -10.4 Because this is a true statement the answer is correct

Ex 4: $4c = -96$

To isolate the "a" move the "4" from where it is to the other side of the equation so that it can be combined with "-96", since "4" and "-96" are like terms.

To move the "4" do the inverse (opposite) of "multiply by 4", which is "divide by 4" to both sides of the equation.

$$\frac{4c = -96}{4} = \frac{-96}{4}$$

c = -24 check you answer by replacing "c" with your answer to see if it is a true statement

CHECK: $4 \cdot -24 = -96$

-96 = -96 Because this is a true statement the answer is correct

Ex 5: $\frac{3}{4}x = 9$

To isolate the "x" the " $\frac{3}{4}$ " needs to move from where it is to the other side of the equation so that it can be combined with "9", since " $\frac{3}{4}$ " and "9" are like terms.

To move the fraction " $\frac{3}{4}$ " multiply both sides of the equation by $\frac{4}{3}$, which is the reciprocal (the flip upside down) of " $\frac{3}{4}$ "

$$\cancel{\frac{4}{3}} \cdot \frac{3}{4} x = 9 \cdot \frac{4}{3} \quad \text{What is left is} \quad x = 9 \cdot \frac{4}{3}$$

REMEMBER: WHEN MULTIPLYING FRACTIONS, MULTIPLY NUMERATOR (TOP NUMBER) BY NUMERATOR AND DENOMINATOR (BOTTOM NUMBER) BY DENOMINATOR

$$x = \frac{36}{3} \quad \text{now divide } 36 \div 3 = 12$$

$$x = 12$$

$$\frac{3}{4}x = 9$$

check your answer by replacing "x" with your answer to see if it is a true statement

CHECK: $\frac{3}{4}(12) = 9$ this is also written as:

$$\frac{3}{4} \cdot \frac{12}{1} = \frac{36}{4} = 9$$

$$9 = 9$$

Because this is a true statement the answer is correct

Practice Problems – Unit 2 – Section 1

For problems 1 – 15, solve each equation. Check your answer.

1) $\frac{y}{5} = 100$

2) $-\frac{c}{7} = 35$

3) $-4 = 7x$ (HINT: you may leave your answer as a fraction)

4) $-3x = -48$

5) $y - 3.7 = 6.93$

6) $p + \frac{1}{4} = -3\frac{1}{2}$ (HINT: you may find it easier to change your fractions to decimals before moving any of the numbers)

7) $0.9x = 11.7$

8) $x + 2.5 = 4.5$

9) $c - 7.88 = 9.24$

10) $b + 5 = -13$

11) $6p = -120$

12) $\frac{3}{4}n = -\frac{3}{8}$

13) $c - 4 = 9$

14) $73.35 = 4.37 + y$

15) $-\frac{r}{3} = -101$

Unit 2 – Solving Equations

Section 2 – Solving Two-Step Equations

(Refer to Pages 100 - 101, problem numbers 1 - 36 in your textbook for additional practice)

Vocabulary and Examples:

Solving Two-Step Equations - solve two-step equations in the same way one-step equations are solved, except two numbers are moved from one side of the equal sign to the other instead of just one. Move one number at a time (for a total of two moves), and **ALWAYS MOVE THE NUMBER NOT ATTACHED TO THE VARIABLE FIRST**, THEN MOVE THE NUMBER ATTACHED TO THE VARIABLE.

$$\text{Ex 1: } 10 = \frac{m}{4} + 2$$

the first number to move (step 1 of the two-step

equation) is 2 because it is the number that is not attached to the m (the 4 is attached to the m) - move the "+2" by doing the inverse (opposite), which is "-2"

$$\begin{array}{r} 10 = \frac{m}{4} + 2 \\ -2 \quad \quad -2 \\ \hline 8 = \frac{m}{4} \end{array}$$

Next, move the 4 (the 2nd step of the two-step equation). Move the "divided by 4" by doing the inverse (opposite), which is "multiply by 4"

$$\begin{array}{r} 8 = \frac{m}{4} \\ \times 4 \quad \quad \times 4 \\ \hline 32 = m \end{array}$$

Check your answer by replacing the "m" in the original problem with your answer to see if it is true.

$$\text{CHECK: } 10 = \frac{32}{4} + 2$$

Make sure to do the order of operation when solving the problem (first divide then add).

$$10 = 8 + 2$$

$$10 = 10$$

Because this is a true statement the answer is correct

$$\text{Ex 2: } -b + 6 = -11$$

NOTICE: the "b" has a negative sign attached to it which means that there is an "-1" in front of "b"

The problem becomes: $-1b + 6 = -11$

$$-1b + 6 = -11$$

First, move the "+6" to the other side of the problem (because it is the number not attached to the b) by doing the inverse of "+6" which is "-6"

$$\begin{array}{r} 1b + 6 = -11 \\ \underline{-6 \quad -6} \\ -1b = -17 \end{array}$$

$$-1b = -17$$

$$\frac{-1b}{-1} = \frac{-17}{-1}$$

$$\mathbf{b = 17}$$

Next, move the "-1" away from the "b" by doing the inverse of "times -1" which is "divide by -1"

Check your answer by replacing the "b" in the original problem with your answer to see if it is true.

$$\text{CHECK: } -1(17) + 6 = -11$$

$$-17 + 6 = -11$$

$$-11 = -11$$

Because this is a true statement the answer is correct

$$\text{Ex 3: } 8 - 3y = 14$$

First, move the "8" to the other side of the equation (because it is not attached to the y) by doing the inverse of "a positive 8" which is "a negative 8, or minus 8"

$$\begin{array}{r} 8 - 3y = 14 \\ \underline{-8 \quad -8} \\ -3y = 6 \end{array}$$

NOTICE: once the "8" has been moved, a "-3y" is left - make sure not to ignore the negative sign because it will make the final answer wrong

$$\frac{-3y}{-3} = \frac{6}{-3}$$

$$\mathbf{y = -2}$$

Next, move the "-3" to the other side of the equation by doing the inverse of "times -3", which is "divide by -3"

$$\text{CHECK: } 8 - 3(-2) = 14$$

$$8 + 6 = 14$$

$$14 = 14$$

Check your answer by replacing the "y" in the original problem with your answer to see if it is true.

Because this is a true statement the answer is correct

Practice Problems – Unit 2 – Section 2

For problems 1 - 11, solve each equation. Check your answer.

1) $-p - 24 = -8$

2) $15 = -z + 8$

3) $\frac{a}{5} + 15 = 30$

4) $10.7 = -d + 4.3$

5) $4x + 92 = 100$

6) $-\frac{1}{5}t - 2 = 4$ (HINT: remember what you learned in section 1 of chapter 2 - multiply both sides by the reciprocal OR you can change your fraction to a decimal and work with it in decimal form)

7) $8 + \frac{c}{-4} = -6$

8) $14 = 6 - 2p$

9) $35 = 3 + 5x$

10) $9 + \frac{n}{5} = 19$

11) $-10 = -6 + 2c$

12) What is the error in the student's work? Solve the equation correctly.

$$\begin{array}{r} 12 - 3y = 15 \\ -12 \quad -12 \\ \hline 3y = 3 \\ \frac{3y}{3} = \frac{3}{3} \\ y = 1 \end{array}$$

13) What is the error in the student's work? Solve the equation correctly.

$$\begin{array}{r} \frac{m}{3} - 9 = -21 \\ +9 \quad +9 \\ \hline \frac{m}{3} = -12 \\ m = -4 \end{array}$$

For problems 14 – 15, define a variable and write an equation for each situation. Then solve.

14) Beneath Earth's surface, the temperature increases 10°C every kilometer. Suppose that the surface temperature is 22°C , and the temperature at the bottom of a gold mine is 45°C . What is the depth of the gold mine?

15) One health insurance policy pays people for claims by multiplying the claim amount by 0.8 and then subtracting \$500. If a person receives a check for \$4650, how much was the claim amount?

Unit 2 – Solving Equations

Section 3 – Solving Multi-Step Equations

(Refer to Page 103, problem numbers 1 - 21 in your textbook for additional practice)

Vocabulary and Examples:

Solving multi-step equations – multi-step equations are solved just like two-step equations. The only difference is that like terms may need to be combined first or the distributive property may need to be used first before terms can be rearranged in the problem to get the variable alone.

Ex 1: $2c + c + 12 = 78$

Notice there are like terms – “2c” and “c” that can be combined. Combine those together first before the 12 is moved. (remember to pay attention to the signs, they determine whether to add or subtract)

$$\begin{array}{r} 2c + c + 12 = 78 \\ \underbrace{\hspace{1.5cm}} \\ 3c + 12 = 78 \\ \quad \underline{-12} \quad \underline{-12} \\ 3c = 66 \end{array}$$

Now the problem becomes a two-step equation (similar to section 2 of Chapter 2) – move the “plus 12” first (it’s not attached to the c) by doing the inverse which is “minus 12”

$$\frac{3c}{3} = \frac{66}{3}$$

Final step – do the inverse of “3 times c” which is “divided by c”

c = 22

CHECK: $2(22) + 22 + 12 = 78$

Check your answer by replacing the “c” in the original problem with your answer to see if it is true.

$$44 + 22 + 12 = 78$$

$$66 + 12 = 78$$

$$78 = 78$$

Because this is a true statement the answer is correct

Ex 2: $-2(b - 4) = 12$

Notice that there are parenthesis in the equation that cannot be solved because a letter and a number CANNOT be subtracted – the DISTRIBUTIVE PROPERTY must be used

$$\begin{array}{c} -2(b - 4) = 12 \\ \curvearrowright \quad \curvearrowright \end{array}$$

REMEMBER: this is the same thing as:
 $-2 \cdot b$ and $-2 \cdot -4$ which equals $-2b + 8$

$$\begin{array}{r} -2b + 8 = 12 \\ \quad \underline{-8} \quad \underline{-8} \\ -2b = 4 \end{array}$$

The problem is now a two-step equation. Move the “plus 8” by doing the inverse which is “minus 8”

$$\begin{array}{r} -2b = 4 \\ \quad \underline{-2} \quad \underline{-2} \\ b = -2 \end{array}$$

The last step is to divide -2 on both sides since that is the inverse of “-2 times b”

CHECK: $-2(-2 - 4) = 12$

Check your answer by replacing the "b" in the original problem with your answer to see if it is true.

$$-2(-6) = 12$$

$$12 = 12$$

Because this is a true statement the answer is correct

Ex 3: $\frac{2x}{3} + \frac{x}{2} = 7$

NOTICE the problem contains fractions –

$$6/1 \cdot 2x/3 + x/2 \cdot 6/1 = 7(6)$$

REMEMBER: fractions cannot be added or subtracted without a common denominator -the easiest way to solve problems that contain fractions is to MULTIPLY both sides by the common denominator (to find the common denominator, multiply the 2 denominators together) - in this case

it would be 6 or $\frac{6}{1}$

REMEMBER: multiply the numerators together and the denominators together. Also, notice that when doing this, the DISTRIBUTIVE PROPERTY is used. This problem becomes:

$$\frac{6 \cdot 2x}{1 \cdot 3} \text{ and } \frac{6 \cdot x}{1 \cdot 2} = 7(6)$$

NOTICE the fractions reduce to whole numbers

$$\begin{array}{r} \frac{12x}{3} + \frac{6x}{2} = 42 \\ \downarrow \quad \downarrow \\ 4x + 3x = 42 \\ \underbrace{\hspace{1.5cm}} \\ 7x = 42 \end{array}$$

Combine like terms 4x and 3x

The last step is to do the inverse of "7 times x" which is "divide by 7"

$$\frac{\cancel{7}x}{\cancel{7}} = \frac{42}{7}$$

$$x = 6$$

CHECK: $\frac{2(6)}{3} + \frac{6}{2} = 7$

Check your answer by replacing the "x" in the original problem with your answer to see if it is true.

$$\frac{12}{3} + 3 = 7$$

$$4 + 3 = 7$$

$$7 = 7$$

Because this is a true statement the answer is correct

Ex 4: $0.5a + 8.75 = 13.25$

Since there are no like terms or parenthesis, begin by moving the numbers from one side of the equation to the other by doing the inverse operation - move the "plus 8.75" by doing the inverse which is "minus 8.75"

$$\begin{array}{r} 0.5a + 8.75 = 13.25 \\ - 8.75 \quad -8.75 \\ \hline \end{array}$$

$$\frac{0.5a}{0.5} = \frac{4.5}{0.5}$$

The last step is to do the inverse of "0.5 times a" which is "divide by a"

a = 9

CHECK: $0.5(9) + 8.75 = 13.25$

Check your answer by replacing the "a" in the original problem with your answer to see if it is true.

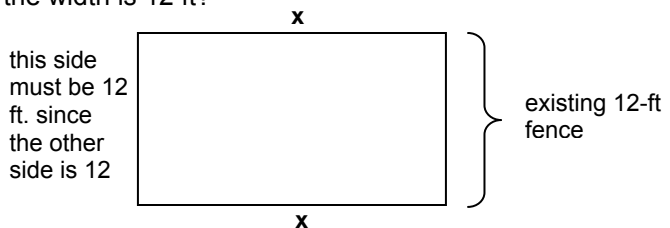
$4.5 + 8.75 = 13.25$

$13.25 = 13.25$

Because this is a true statement the answer is correct

Ex 5: A gardener is planning a rectangular garden area in a community garden. His garden will be next to an existing 12-ft fence. The gardener has a total of 44 ft of fencing to build the other three sides of his garden. How long will the garden be if the width is 12 ft?

Since the problem is talking about 44 ft of fencing to go around the 3 sides, it is referring to perimeter (minus the 12 ft side that already has the 12-ft fence)



The length of the remaining 3 sides are:

length of side	plus	12 ft	plus	length of side	=	amount of fencing
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x	+	12 ft	+	x	=	44
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Combine like terms x and x

$$x + 12 + x = 44$$

↓ ↓
2x

Move the 12 to the other side of the equation by doing the inverse of "plus 12" which is "minus 12"

The last step is to do the inverse of "2 times x" which is "divide by 2"

$$\begin{array}{r} 2x + 12 = 44 \\ - 12 \quad -12 \\ \hline 2x = 32 \\ \frac{2x}{2} = \frac{32}{2} \\ x = 16 \end{array}$$

CHECK:

$$16 + 12 + 16 = 44$$

$$44 = 44$$

Because this is a true statement the answer is correct

Steps for Solving a Multi-Step Equation

Step 1 – Clear the equation of fractions

Step 2 – Use the Distributive Property to remove parenthesis on each side

Step 3 – Combine like terms on each side

Step 4 – Undo any addition or subtraction

Step 5 – Undo multiplication or division

Practice Problems – Unit 2 – Section 3

For problems 1 - 12, solve each equation.

1) $2(8 + p) = 22$

2) $15 = -3(2d - 1)$

3) $-(g + 5) = -14$ (HINT: remember that there is an invisible -1 in front of the parenthesis that must be distributed through the parenthesis)

4) $8y - (2y - 3) = 9$ - Round your answer to the hundredths place. (HINT: remember that there is an invisible -1 in front of the parenthesis that must be distributed through the parenthesis)

5) $72 + 4 - 14c = 36$

6) $7m - 3m - 6 = 6$

7) $\frac{1}{5} + \frac{3w}{15} = \frac{4}{5}$

8) $3.5 = 12d - 5d$

9) $1.025x + 2.458 = 7.583$

10) $26.54 - p = 0.5(50 - p)$

11) $4 + \frac{m}{8} = \frac{3}{4}$

12) $1.12 + 1.25y = 8.62$

13) **Write an equation to model each situation, then solve your equation.**

You are fencing a rectangular puppy kennel with 25 ft of fence. The side of the kennel against your house does not need a fence. This side is 9 ft long. Find the dimensions of the kennel

14) Explain the error in the student's work at the right.

$$\begin{aligned} \frac{3}{8}x - 1 &= 4 \\ (8)\left(\frac{3}{8}x - 1\right) &= 4(8) \\ 3x - \cancel{1} &= 32 \\ + \cancel{1} + 1 & \\ \hline 3x &= 33 \\ \hline 3 & \quad 3 \\ \hline x &= 11 \end{aligned}$$

15) Suppose you want to solve the equation $-3m + 4 + 5m = -6$. What step would you do as your first step?

Unit 2 – Solving Equations

Section 4 – Equations with Variables on Both Sides

(Refer to Pages 189 - 190, problem numbers 1 - 67 in your textbook for additional practice)

Vocabulary and Examples:

To solve equations with variables on both sides - Remember to do the distributive property and combine all like terms first. Also, make sure to move all of the variables to one side of the equation and all of the numbers to the other.

$$\text{Ex 1: } 6a + 3 = 8a - 21$$

NOTICE: there are “a’s” on both sides of the equation (“6a” and “8a”) and numbers on both sides of the equation (“3” and “-21”)

Rearrange the problem so that the “6a” and “8a” are on one side of the equation and the “3” and “-21” are on the other side of the equation. Remember that to move a number or variable from one side of the equation to the other, do the inverse of the operation

$$\begin{array}{r} 6a + 3 = 8a - 21 \\ -8a \quad -8a \\ \hline -2a + 3 = -21 \end{array}$$

Either move the “6a” over to join the “8a”, the “8a” over to join the “6a”, the “3” over to join the “-21” or the “-21” over to join the “3”.

What is chosen to move first doesn’t matter. Typically, it is best to put the variable to the left of the equal sign and the numbers to the right of the equal sign.

In this case, the “8a” moved to join the “6a” so that the variables are to the left of the equal sign. Do the inverse of “positive (or plus) 8a” which is “negative or minus 8a” – make sure to pay attention to the signs. Notice that what is now left to the right side of the equation is “-21”

$$\begin{array}{r} -2a + 3 = -21 \\ -3 \quad -3 \\ \hline -2a = -24 \\ -2 \quad -2 \\ \hline \end{array}$$

Next, bring the “+3” over to join the “-21” by doing the inverse of “+3” which is “minus 3”

Last, bring the “-2” over to join the “-24” by doing the inverse of “-2 times a” which is “divide by -2”

$$a = 12$$

$$\text{CHECK: } 6(12) + 3 = 8(12) - 21$$

$$72 + 3 = 96 - 21$$

$$75 = 75$$

Because this is a true statement the answer is correct

Ex 4: $6m - 5 = 7m + 7 - m$

understood -1 in front of the "m"

$$6m - 5 = 7m + 7 - m$$

\swarrow \searrow
 $6m$

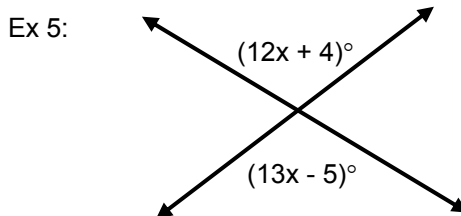
$$\begin{array}{r} 6m - 5 = 6m + 7 \\ \underline{-6m} \quad \underline{-6m} \\ -5 = 7 \end{array}$$

NOTICE that there are like terms of "6m" on the left side of the equal sign and "7m" and "-m" on the right side of the equal sign. Combine the "7m" and the "-m" together first.

Next, combine either the "m's" or the numbers together. In this case the "6m" and the "6m" are being combined by doing the inverse, which is "- 6m"

NOTICE that the "6m" canceled out of both sides of the equation. What is left is " $-5 = 7$ " which is a false statement - 5 does not equal 7.

A false statement means that no matter what value is plugged in for the variable, it will ALWAYS BE A FALSE STATEMENT. THERE IS NO VALUE FOR "M" THAT WILL MAKE THIS TRUE! Therefore, the final answer is, **NO SOLUTION**



Find the value of x

In this type of problem, notice that the two angles are across (or opposite) from each other. These are called **vertical angles** and vertical angles are always equal. Since that is true, set these two angle measures equal to each other, and solve for x

$$12x + 4 = 13x - 5$$

$$\begin{array}{r} 12x + 4 = 13x - 5 \\ \underline{-13x} \quad \underline{-13x} \end{array}$$

$$\begin{array}{r} -1x + 4 = -5 \\ \underline{-4} \quad \underline{-4} \end{array}$$

$$\begin{array}{r} -1x = -9 \\ \underline{-1} \quad \underline{-1} \end{array}$$

$$x = 9$$

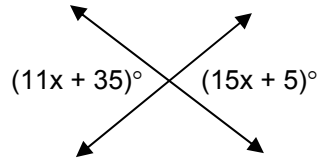
Rearrange the problem to get the like terms together on the same side of the equation. Remember to do the inverse operation to move terms from one side of the equation to the other.

CHECK: $12(9) + 4 = 13(9) - 5$
 $108 + 4 = 117 - 5$
 $112 = 112$

Because this is a true statement the answer is correct

Practice Problems – Unit 2 – Section 4

1) Find the value of x



For problems 2 - 11, solve each equation. Check your answers. If the equation has an answer of identity, then write *identity*, if it has an answer of no solution, then write *no solution*.

2) $5m + 3 = 3m + 9$

3) $6b + 14 = -7 - b$

4) $4p - 10 = p + 3p - 2p$

5) $5m - 2(m + 2) = -(2m + 15)$ HINT: the 2 that needs to be distributed is a “negative 2” and there is an invisible -1 that needs to be distributed on the right side of the equal sign.

6) $6x = 4(x + 5)$

7) $18x - 5 = 3(6x - 2)$

8) $3(x - 4) = 3x - 12$

9) $9x + 3x - 10 = 3(3x + x)$

10) $\frac{7}{8}w = \frac{4}{8}w + \frac{6}{8}w$

11) $6(6g - 2) + 8(1 - 5g) = 2g$

For problems 12 – 15, write and solve an equation for each situation. Check the reasonableness of your answer.

12) One telephone company charges \$16.95 per month and \$.05 per minute for local calls. Another company charges \$22.95 per month and \$.02 per minute for local calls. For what number of minutes of local calls per month is the cost of the plans the same?

13) One health club charges a \$44 sign-up fee and \$30 per month. Another health club charges a \$99 sign-up fee and \$25 per month. For what number of months is the cost the same?

14) A toy company spends \$1500 each day for factory expenses plus \$8 per teddy bear. How many bears must the company sell in one day to equal its daily costs. Write an equation and solve.

15) A company manufactures tote bags. The company spends \$1200 each day for overhead expenses plus \$9 per tote bag for labor and materials. The tote bags sell for \$25 each. How many tote bags must the company sell each day to equal its daily costs for overhead, labor, and materials? Write an equation and solve.

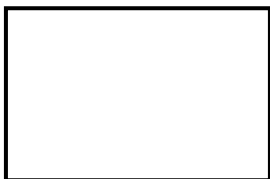
Unit 2 – Solving Equations

Section 5 – Equations and Problem Solving

(Refer to Pages 193 - 194, problem numbers 1 - 51 in your textbook for additional practice)

Vocabulary and Examples:

Ex 1: The length of a rectangle is 6 in. more than its width. The perimeter of the rectangle is 24 in. What is the length of the rectangle?



length = $w + 6$

because the problem tells us that the length is 6 in. more than the width (which we know is w)

width = w (because the width value is not given in the problem)

perimeter – the formula for perimeter (the distance around the outside of the rectangle) is to add all of the sides -
 $\text{length} + \text{width} + \text{length} + \text{width} = \text{perimeter}$

$$2l + 2w = 24$$

Since the length is " $w + 6$ ", replace the " l " in the above equation with " $w + 6$ "

$$2(w + 6) + 2w = 24$$

distribute the 2 to the " w " and " $+6$ "

$$2(w + 6) + 2w = 24$$

combine like terms " $2w$ " and " $+2w$ "

$$2w + 12 + 2w = 24$$

subtract 12 from both sides

$$4w + \cancel{12} = 24$$

$$\underline{- \cancel{12} - 12}$$

divide both sides by 4

$$\frac{4w}{4} = \frac{12}{4}$$

$$w = 3$$

CHECK: $2(3 + 6) + 2(3) = 24$

$$2(\cancel{3+6}) + 2(3) = 24$$

$$2(\cancel{9}) + 2(\cancel{3}) = 24$$

$$18 + 6 = 24$$

$24 = 24$ Because this is a true statement the answer is correct

Ex 2: the sum of three consecutive integers is 147. Find the integers.

Remember that sum means to add, and consecutive means one right after the other. For example, 60 and 61 are consecutive integers because they follow each other on a number line. Consecutive even integers would be numbers like 2 and 4, (they are even numbers that are consecutive because they follow each other on the number line). Consecutive odd integers would be numbers like 131 and 133 for the same reason.

Example 2 is asking for three numbers that are consecutive and when added together will equal 147.

Let x = the first unknown number, which means that " $x + 1$ " will be the second unknown number and " $x + 2$ " will be the third unknown number. When added together, they should all equal 147.

$$x + (x + 1) + (x + 2) = 147 \quad \text{- combine like terms (all of the "x's" and all of the numbers)}$$

$$\begin{array}{r} 3x + \cancel{3} = 147 \\ -\cancel{3} \quad -3 \\ \hline \end{array} \quad \text{- subtract 3 from both sides}$$

$$\frac{3x}{3} = \frac{147}{3} \quad \text{- divide both sides by 3}$$

$$x = 48$$

since $x = 48$, then $x + 1$ will be 49, and $x + 2$ will be 50
so the 3 consecutive numbers are:
48, 49, and 50

CHECK: $48 + 49 + 50 = 147$
 $147 = 147$

Because this is a true statement the answer is correct

Ex 3: A train leaves a station at 1 P.M. It travels at an average rate of 60 mph. A high-speed train leaves the same station an hour later. It travels at an average rate of 96 mph. The second train follows the same route as the first train on a track parallel to the first. In how many hours will the second train catch up with the first train?

To solve problems that deal with distance, rate and time, use a table like the one below.

The formula for these kinds of problems is: rate x time = distance

Topic	Rate	\times	Time	$=$	Distance

Topic	Rate	x	Time	=	Distance

Put the topic that is being discussed in the problem in this column. In this case, it could be train 1 and train 2.

There will always be 2 topics being discussed in the problem

Train	Rate	x	Time	=	Distance
1					
2					

Put the rate the object is going in this column. In this case, it would be 60 mph and 96 mph

Train	Rate	x	Time	=	Distance
1	60				
2	96				

In this column, put how much time it takes the object to go the distance (if the problem gives you this information)

In this case, the problem does not tell you.

So, it will be "t" for time for train 1, and it will be "t - 1" for train 2 because the problem says that the second train left 1 hour later.

Train	Rate	x	Time	=	Distance
1	60		t		
2	96		(t - 1)		

REMEMBER: the equation is: **rate x time = distance** – once the table is filled in, multiply the rate and the time and put the answer in the distance column. Sometimes, like in the case of train 2, the distributive property will have to be used.

Train	Rate	x	Time	=	Distance
1	60		t		60t
2	96		(t - 1)		96t - 96

In this column put the answer to the rate and time being multiplied

$$60 \cdot t = 60t \text{ and}$$

$$96(t - 1) = 96t - 96$$

HOW TO SET UP THE PROBLEM - Once the table is completely filled in, there are two possible ways to set up the problem in order to solve it.

IF THE ORIGINAL WORD PROBLEM GIVES THE TOTAL DISTANCE TRAVELED, add the two answers in the distance column of your table and set it equal to the total distance given in the original word problem.

IF THE ORIGINAL WORD PROBLEM DOES NOT GIVE THE TOTAL DISTANCE TRAVELED, set the two answers in the distance column of your table equal to each other.

In the case of this example, the total distance is not given in the original word problem, so set **60t** and **96t - 96** equal to each other.

Subtract 96t from both sides

$$\begin{array}{r} 60t = 96t - 96 \\ -96t \quad -96t \\ \hline \end{array}$$

Divide both sides by -36

$$\begin{array}{r} -36t = -96 \\ -36 \quad -36 \\ \hline \end{array}$$

$$t = 2\frac{2}{3}$$

It took train one (which is t) $2\frac{2}{3}$ hours. The question specifically asked how long it would take train two to catch up to train one. Replace “ t ” in your table for train two with $2\frac{2}{3}$, which becomes $2\frac{2}{3} - 1$. Subtract the two numbers. It takes train two $1\frac{2}{3}$ hours to catch up to train one.

Ex 4: Terry drives into the city to buy a software program at a computer store. Because of traffic conditions, he averages only 15 mph. On his drive home, he averages 35 mph. If the total travel time is 2 hours, how long will it take him to drive to the computer store?

Fill in the table with all of the values from the word problem. The two topics being discussed are “to the computer store” and the “return trip home”

Terry’s trip	Rate	x Time	=	Distance
To the computer store				
Return home				

Fill in the value for the rate going to the store (15 mph) and the rate coming home (35 mph)

Terry’s trip	Rate	x Time	=	Distance
To the computer store	15			
Return home	35			

The problem show that the total trip took 2 hours. So if it took “ t ” hours to get to the computer store, than the remainder of the 2 hours is what it took to get back home (or $2 - t$)

Terry’s trip	Rate	x Time	=	Distance
To the computer store	15	t		
Return home	35	$(2 - t)$		

Multiply the rate and the distance for going to the computer store and coming back home. The distance column will be “15t” and “70 – 35t”

Terry's trip	Rate	x Time	= Distance
To the computer store	15	t	15t
Return home	35	(2 – t)	70 - 35t

Since the problem does not give a total distance traveled, set the two answers in the distance column equal to each other.

$$\begin{array}{r}
 15t = 70 - 35t \\
 + 35t \quad \quad + 35t \\
 \hline
 50t = 70 \\
 \hline
 50 \quad \quad 50 \\
 \hline
 t = 1.4
 \end{array}$$

Since the question specifically asked for the time it would take to go to the computer store (“t”), the answer is 1.4 hours.

Ex 5: Jane and Peter leave their home traveling in opposite directions on a straight road. Peter drives 15 mph faster than Jane. After 3 hours, they are 225 miles apart. Find Peter's rate and Jane's rate.

Fill in the table with all of the values from the word problem. The two topics being discussed are “what Jane is doing” and “what Peter is doing”

Person	Rate	x Time	= Distance
Jane			
Peter			

Practice Problems – Unit 2 – Section 5

- 1) The length of a rectangle is 3 in. more than its width. The perimeter of the rectangle is 30 in.
 - a) Define a variable for the width.
 - b) Write an expression for the length in terms of the width.
 - c) Write an equation to find the width of the rectangle. Solve your equation.
 - d) What is the length of the rectangle?

- 2) The width of the rectangle is one half its length. The perimeter of the rectangle is 24 in. What are the width and length of the rectangle?

- 3) The sum of the two consecutive integers is -35. If n = the first integer, which equation best models the situation?

a) $n(n + 1) = -35$	b) $n + 2n = -35$
c) $n + (n + 1) = -35$	d) $n + (2n + 1) = -35$

- 4) The sum of two consecutive odd integers is 56.
 - a) Define a variable for the smaller integer.
 - b) What must you add to an odd integer to get the next greater odd integer?
 - c) Write an expression for the second integer.
 - d) Write and solve an equation to find the two odd integers.

- 5) The sum of three consecutive integers is 915. What are the integers?

- 6) A moving van leaves a house traveling at an average rate of 35 mph. The family leaves the house $\frac{3}{4}$ hour later following the same route in a car. They travel at an average rate of 50 mph.
 - a) Define a variable for the time traveled by the moving van.
 - b) Write an expression for the time traveled by the car.
 - c) Copy and complete the table.

Vehicle	Rate	x	Time	=	Distance Traveled
Van					
Car					

- d) Write and solve an equation to find out how long it will take the car to catch up with the moving van.

- 7) Juan drives to work. Because of traffic conditions, he averages 22 mph. He returns home averaging 32 mph. The total travel time is $2\frac{1}{4}$ hours.
 - a) Define a variable for the time Juan takes to travel to work. Write an expression for the time Juan takes to return home.
 - b) Write and solve an equation to find the time Juan spends driving to work.

- 8) The tail of a kite is 1.5 ft. plus twice the length of the kite. Together, the kite and tail are 15 ft 6 in. long.
 - a) Write an expression for the length of the kite and tail together.
 - b) Write 15 ft. 6 in. in terms of feet.
 - c) Write and solve an equation to find the length of the tail.

9) Ellen and Kate race on their bicycles to the library after school. They both left school at 3:00 P.M. and bicycled along the same path. Ellen rode at a speed of 12 mph and Kate rode at 9 mph. Ellen got to the library 15 minutes before Kate.

- a) How long did it take Ellen to get to the library?
- b) At what time did Ellen get to the library?

10) Two airplanes depart from an airport traveling in opposite directions. The second airplane is 200 mph faster than the first. After 2 hours they are 1100 miles apart. Find the speed of the airplanes.

11) Three friends were born in consecutive years. The sum of their birth years is 5961. Find the year in which each person was born

12) Two boats leave a ramp traveling in opposite directions. The second boat is 10 mph faster than the first. After 3 hours they are 150 miles apart. Find the speeds of the boats.

Unit 2 – Solving Equations

Section 6 – Formulas

(Refer to Page 199, problem numbers 1 - 28 in your textbook for additional practice)

Vocabulary and Examples:

Literal equation – an equation involving two or more variables. Formulas are a type of literal equation. To solve a literal equation, get the variable the problem is asking to solve for alone on one side of the equation, and move everything else to the other side of the equation.

Ex 1: $A = \frac{1}{2}bh$ solve the formula for “h” - this means the “h” is going to be alone on one side of the equation, and all of the other terms must move to the other side of the equation. Remember to do the inverse operation.

To get the “h” alone, first move the “ $\frac{1}{2}$ ” by doing the inverse operation of multiplying a fraction which is to multiply by the reciprocal (which is $\frac{2}{1}$ or 2) on both sides of the equation.

$$(2)A = \cancel{(2)}\frac{1}{\cancel{2}}bh$$

$$\frac{2A}{b} = \frac{\cancel{b}h}{\cancel{b}}$$

$$\frac{2A}{b} = h$$

Next, divide both sides by b

the “h” is now alone

Ex 2: solve $y = 5x + 7$ for x

get the “x” alone by first moving the 7 by doing the inverse of “+7” which is “-7”

$$\begin{array}{r} y = 5x + 7 \\ -7 \quad -7 \\ \hline y - 7 = \frac{5x}{5} \end{array}$$

next, divide both sides by 5

$$\frac{y-7}{5} = x$$

Practice Problems – Unit 2 – Section 6

For problems 1 - 10, solve each formula for the given variable.

1) $y + 2x = 5$ solve for y

2) $5x + 4y = 4$ solve for y

3) $dx = c$ solve for x

4) $z - a = y$ solve for z

5) $S = C + rC$ solve for r

5) $\frac{y-b}{m} = x$ solve for y

6) $V = lwh$ solve for h

7) $ax + by = c$ solve for y

8) $y = 3(w - y)$ solve for y (HINT: you must use the distributive property first)

9) $2x + 10 = 5y - 4$ solve for y

10) $\frac{a}{b} = \frac{c}{d}$ solve for b (HINT: you must move the b to the other side of the equation first, then move the “ c ” and “ d ” to the other side of the equation.)

11) Bricklayers use the formula $N = 7LH$ to estimate the number of bricks “ N ” needed to build a wall of height “ H ” and length “ L ”

a) Solve the equation for “ H ”

b) What is the height of a wall that is 30 feet long and that requires 2310 bricks to build?

12) You can use the formula $a = \frac{h}{n}$ to find the batting average “ a ” of a batter who has “ h ” hits in “ n ” times at bat.

a) Solve the equation for “ h ”

b) If a batter has a batting average of .265 and has been at bat 200 times, how many hits does the batter have?

13) a) The formula $I = prt$ gives the amount of simple interest “ I ” earned by principal “ p ” at an annual interest rate “ r ” over “ t ” years. Solve this formula for “ p ”

b) Find “ p ” if $r = 0.035$, $t = 4$, and $I = \$420$.

Unit 2 – Solving Equations

Section 7 – Using Measures of Central Tendency

(Refer to Pages 606 - 607, problem numbers 1 – 16 and Pages 611 – 613, problem numbers 1 - 20 in your textbook for additional practice)

Vocabulary and Examples:

Measures of Central Tendency – mean, median, and mode

Mean - $\frac{\text{sum of the data items}}{\text{total number of data items}}$

Use the mean to describe the middle of a set of data that **does not** have an outlier. An outlier is a data value that is much higher or lower than the other data values in the set. The mean is often referred to as the average.

Median – is the middle value in the set of data when the numbers are arranged in order from least to greatest. For a set of data containing an even number of data items, the median is the mean of the two middle data items.

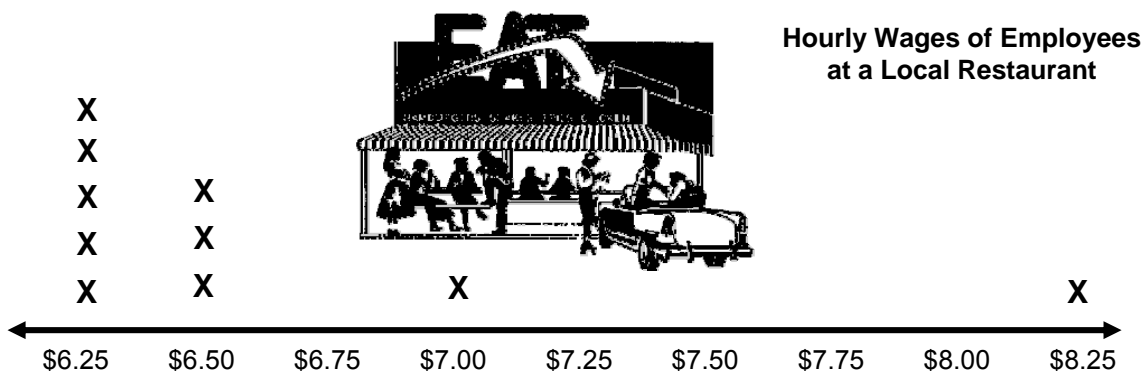
Use the median to describe the middle of a set of data that **does** have an outlier.

Mode – is the data item that occurs the most times. It is possible for a set of data to have no mode, one mode, or more than one mode.

Use the mode when the data are nonnumeric or when choosing the most popular item.

Range – the largest data item minus the smallest data item

Ex 1: Find the mean, median and mode of the data in the line plot below.



Mean: $\frac{6.25 + 6.25 + 6.25 + 6.25 + 6.25 + 6.50 + 6.50 + 6.50 + 7.00 + 8.25}{10} = \frac{66}{10} = 6.6$

↓
total number of employees

Median: 6.25 6.25 6.25 6.25 6.25 6.50 6.50 6.50 7.00 8.25

2 middle values = $\frac{6.25 + 6.50}{2} = \frac{12.75}{2} = 6.375$

Mode: 6.25

Ex 2: Suppose your grades on 3 history exams are 80, 93, and 91. What grade do you need on your next exam to have a 90 average on the four exams?

Mean (average): $\frac{80+93+91+x}{4} = 90$

Annotations:
- A bracket above 80+93+91+x is labeled "3 test scores given in problem".
- An arrow points from x to "unknown test score".
- An arrow points from the equals sign and 90 to "total average to achieve".
- An arrow points from the denominator 4 to "total number of test scores".

$$\frac{264+x}{4} = 90 \quad \text{combine like terms}$$

$$\frac{264+x}{4} = 90 \quad \text{multiply both sides by 4}$$

~~4~~ x 4 ~~x~~ x 4

$$\begin{array}{r} 264 + x = 360 \\ -264 \quad -264 \\ \hline \end{array} \quad \text{Subtract 264 from both sides}$$

$$x = 96$$

You must score a 96 on your next exam to receive an average score of 90 in the class

Stem-and-leaf plot – a way to organize data. The data is separated into a stem (**ALL OF THE DIGITS TO THE LEFT OF THE LAST DIGIT**) and a leaf (**ONLY THE LAST DIGIT**)

Ex 3: Make a stem-and-leaf plot for the list of data:

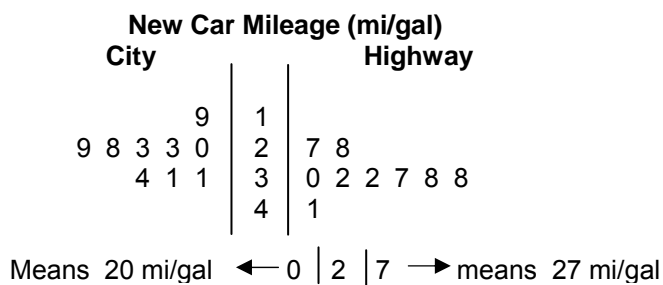
15, 15, 9, 9, 20, 23, 7, 17, 6

0		6 7 9 9
1		5 5 7
2		0 3

1 | 6 means 16

Make sure to include the key at the bottom of any stem-and-leaf plot created

Ex 3: Find the mean of the city mileage and highway mileage for nine new cars



This stem-and-leaf plot is actually a double one. The numbers to the left represent the city miles and the numbers to the right represent the highway miles. The city miles are: 19, 20, 23, 23, 28, 29, 31, 31, and 34. The highway miles are: 27, 28, 30, 32, 32, 37, 38, 38, and 41.

Mean city mileage:
$$\frac{19 + 20 + 23 + 23 + 28 + 29 + 31 + 31 + 34}{9} = 26.\overline{4} \text{ mi/gal}$$

Mean highway mileage:
$$\frac{27 + 28 + 30 + 32 + 32 + 37 + 38 + 38 + 41}{9} = 33.\overline{6} \text{ mi/gal}$$

Practice Problems – Unit 2 – Section 7

For problems 1 - 2, find the mean, median, and mode.

1) weight of textbooks in ounces: 12 10 9 15 16 10

2) weights of channel catfish in pounds: 4.8 5 2.3 4.5 4.8 5.2

For problems 3 – 4, write and solve an equation to find the value of “x”

3) 100 121 105 113 108 x mean = 112

4) 31.7 42.8 26.4 x mean = 35

For problems 5 – 7, find the range.

5) 5.3 6.2 3.1 4.8 7.3

6) -12 -15 5 3 -2 0 -7

7) $2\frac{1}{2}$ $3\frac{1}{3}$ $-5\frac{3}{4}$ $\frac{3}{8}$ $3\frac{5}{8}$

For problems 8 -10, make a stem-and-leaf-plot for each set of data. Don't forget to include the key at the bottom of the stem-and-leaf plot.

8) 18 35 28 15 36 10 25 22 15

9) 18.6 18.4 17.6 15.7 15.3 17.5

10) 785 776 788 761 768 768 785

For problems 11 – 12, find the mean, median, mode, and range of EACH SIDE of the stem-and-leaf plot.

11)

Time Spent on Homework (minutes/day)	
Class A	Class B
6 6 4 3	4 1 1 4 5 7
9 8 6 4 4 4	5 0 2 2 2 4
5 2 1 0	6 5 8 9
8 7 6 6 4 2	7 3 6 7 9 9 9
means 43 ← 3 4 1 → means 41	

12)

Growth of 2 Varieties of Tulip Plants (inches/day)	
Type A	Type B
6 3 3	2
3 2 1 1	3 1 1 2
1	4 3 5 8
	5 2 4
means 0.33 ← 3 3 1 → means 0.31	

Unit 3 – Solving Inequalities

Section 1 – Inequalities and Their Graphs

(Refer to Page 206, problem numbers 1-36 in your textbook for additional practice)

Vocabulary and Examples:

Inequality – the math symbols: $<$ which means “is less than”, $>$ which means “is greater than”, \leq which means “is less than or equal to”, and \geq which means “is greater than or equal to”

Solution of an Inequality – any number that makes the inequality true. Because the solution to an inequality has an infinite amount of solutions, the solutions are graphed on a number line.

Ex 1: $x < 3$ the solution to this inequality is any number that is less than 3.

Ex 2: is each number a solution of: $x \leq 7$? (a) 9 (b) -1 (c) $\frac{14}{2}$

Substitute the values in for “x” and determine which are true statements.

$9 \leq 7$ false $-1 \leq 7$ true $\frac{14}{2}$ which is really 7 $7 \leq 7$ true

The correct answers are (b) and (c)

Ex 3: Is each number a solution of: $2 - 5x > 13$ (a) 3 (b) -4

Substitute the values in for “x” and determine which are true statements.

(a) $2 - 5(3) > 13$

(b) $2 - 5(-4) > 13$

$2 - 15 > 13$

$2 + 20 > 13$

$13 > 13$ false

$22 > 13$ true

Graphing Inequalities – When graphing inequalities, the inequality symbol determines whether the circle above the value is closed (colored in) or open (not colored in).

$<$ or $>$ open circle (not colored in) \circ means that the number this circle is above is NOT included as part of the solution to the problem

\leq or \geq closed circle (colored in) \bullet means that the number this circle is above IS included as part of the solution to the problem

Ex 4: Graph $c > -2$ this reads as: “c is greater than -2”. Any number greater than (“bigger than”) -2 is a solution to this inequality.

HINT: as long as the variable is first, the inequality will point in the direction that the arrow should go on the graph. In this example, it is an open circle and the arrow should point to the right

