Unit 3 – Solving Inequalities

Section 1 – Inequalities and Their Graphs

(Refer to Page 206, problem numbers1 36 in your textbook for additional practice)

Vocabulary and Examples:

Inequality – the math symbols: $<$ which means "is less than", $>$ which means "is greater than", $\leq$ which means "is less than or equal to", and $\geq$ which means "is greater than or equal to"

Solution of an Inequality – any number that makes the inequality true. Because the solution to an inequality has an infinite amount of solutions, the solutions are graphed on a number line.

Ex 1: $x < 3$ the solution to this inequality is any number that is less than 3.

Ex 2: is each number a solution of: $x \leq 7$? (a) 9 (b) -1 (c) $\frac{14}{2}$

Substitute the values in for "x" and determine which are true statements.

$9 \leq 7$ false $-1 \leq 7$ true $\frac{14}{2}$ which is really 7 $7 \leq 7$ true

The correct answers are (b) and (c)

Ex 3: Is each number a solution of: $2 - 5x > 13$ (a) 3 (b) -4

Substitute the values in for "x" and determine which are true statements.

(a) $2 - 5(3) > 13$

$2 - 15 > 13$

$13 > 13$ false

(b) $2 - 5(-4) > 13$

$2 + 20 > 13$

$22 > 13$ true

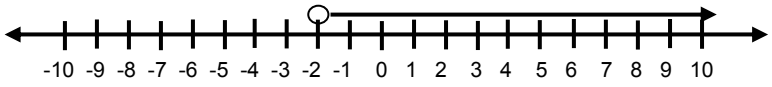
Graphing Inequalities – When graphing inequalities, the inequality symbol determines whether the circle above the value is closed (colored in) or open (not colored in).

$<$ or $>$ open circle (not colored in) ○ means that the number this circle is above is NOT included as part of the solution to the problem

$\leq$ or $\geq$ closed circle (colored in) ● means that the number this circle is above IS included as part of the solution to the problem

Ex 4: Graph $c > -2$ this reads as: " c is greater than -2". Any number greater than ("bigger than") -2 is a solution to this inequality.

HINT: as long as the variable is first, the inequality will point in the direction that the arrow should go on the graph. In this example, it is an open circle and the arrow should point to the right



Ex 5: Graph $4 \geq m$ notice that the variable is not first, so flipping the entire problem so that the variable is first will make it so that the rule of "as long as the variable is first, the inequality will point in the direction that the arrow should go on the graph" is true.

$m \leq 4$ notice that the inequality also flips directions.

In this example, it is a closed circle and the arrow should point to the left

Ex 6: write an inequality for each graph

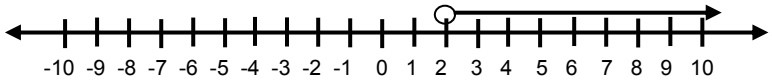
First, pick any variable. Then, notice the direction of the arrow and whether the circle is closed (colored in) open (not colored in). Last, name the number

a)

variable - "a"
arrow is pointing left and closed circle – means "less than or equal to"
number - "-4"

THEREFORE: $a \leq -4$

b)



variable - "c"
arrow is pointing right and open circle – means "greater than"
number - "2"

THEREFORE: $c > 2$

Practice Problems – Unit 3 – Section 1

For problems 1 and 2, determine if the number following the inequality is a solution of the given inequality.

1) $0.5 > c$; 4

2) $d \leq \frac{17}{3}$; 5

For problems 3 – 6, is each number a solution of the given inequality?

3) $3x - 7 > -1$	a. 2	b. 0	c. 5
4) $\frac{4-m}{m} \geq 5$	a. 0.5	b. 2	c. -4
5) $5(2q - 8) \geq 7$	a. -2	b. $\frac{9}{2}$	c. 6
6) $2y + 1 < -3$	a. 0	b. -2	c. 1

For problems 7 - 9, graph each inequality.

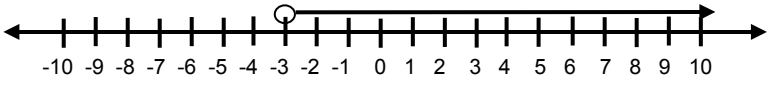
7) $x < 4$

8) $x \geq 6$

9) $4.25 > c$

For problems 10 – 13, write an inequality for each graph.

10)



11)

12)

13)

For problems 14 – 15, write each inequality in words.

14) $a \leq 3$

15) $35 \geq w$

Unit 3 – Solving Inequalities

Section 2 – Solving Inequalities Using Addition and Subtraction

(Refer to Page 206, problem numbers 1 - 15 in your textbook for additional practice)

Vocabulary and Examples:

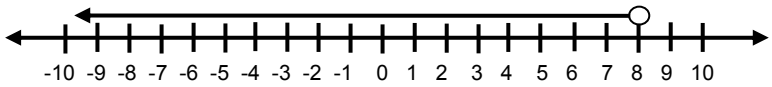
To solve inequalities using multi-steps - these problems are solved exactly like multi-step equations, the only difference is instead of an equal sign in the problem, there is an inequality AND most likely, the answer will be graphed on a number line.

Ex 1: solve $x - 3 < 5$ check and graph your solution

$x - 3 < 5$ add 3 to both sides

$+3 \quad +3$

$x < 8$ graph this on a number line



CHECK: replace "x" with any number less than 8 (like 5 for example)

$5 - 3 < 5$

$2 < 5$ true

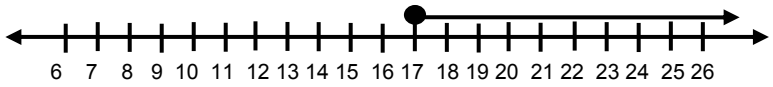
Ex 2: Solve $12 \leq x - 5$ graph and check your solution

$12 \leq x - 5$ add 5 to both sides

$+5 \quad +5$

$17 \leq x$ notice that the variable is not first, so flipping the entire problem so that the variable is first will make it so that the rule of "as long as the variable is first, the inequality will point in the direction that the arrow should go on the graph" is true.

$x \geq 17$



CHECK: replace "x" with any number that is "greater than or equal to" 17 (for example: 20

$12 \leq 20 - 5$

$12 \leq 15$ true

Practice Problems – Unit 3 – Section 2

For problems 1 -6 , solve and graph each inequality.

1) $x - 1 > 10$ 2) $-5 \geq b - 1$ 3) $y - \frac{1}{2} \leq -5$ 4) $-3.4 > m - 1.8$

5) $n - 2\frac{1}{2} < \frac{1}{2}$ 6) $k + 3 \leq 4$

For questions 6 - 10, solve each inequality.

7) $w - 3 + 1 \geq 9$ 8) $-7.7 \geq x - 2$ 9) $5.3 > 1.6 + n - 2.3$

10) $5 \leq v - 4 - 7$

11) Your brother has $2000 saved for a vacation. His airplane ticket is $637. Write and solve an inequality to find how much he can spend for everything else.

12) You have an allowance of $15.00 per week. You are in a bowling league that costs $6.50 each week, and you save at least $5.00 each week. Write and solve an inequality to show how much you have left to spend each week.

13) $-3d + 4(d + 3) > 4$ (HINT: make sure to do the distributive property first)

14) $5(y - 2) - 4(y - 1) < 0$ (HINT: make sure to do the distributive property through both sets of parenthesis)

Unit 3 – Solving Inequalities

Section 3 – Solving Inequalities Using Multiplication and Division

(Refer to Page 206, problem numbers 16 - 36 in your textbook for additional practice)

Vocabulary and Examples:

To solve inequalities using Multiplication and Division - these problems are solved exactly like multi-step equations, the difference is that instead of an equal sign in the problem, there is an inequality AND most likely, the answer will be graphed on a number line.

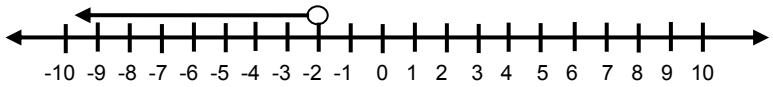
IMPORTANT - if the number being moved is either multiplied or divided AND is NEGATIVE, the inequality in the final answer MUST BE REVERSED IN DIRECTION

Ex 1: $\frac{x}{2} < -1$ multiply each side by 2

$\frac{x}{2} < -1$

$x \cdot 2 \quad \underline{x\ 2}$

$x < -2$



Ex 2: $-\frac{2}{3}n \le 2$

Method 1: Since the entire fraction is negative, the negative sign can be assigned to either the 2 or the 3. Once this is done, the problem can be solved as a two-step problem

$\frac{-2}{3}n \le 2$ OR $\frac{2}{-3}n \le 2$

After placing the negative sign with either the numerator or denominator, move the denominator by multiplying both sides of the inequality by 3 or -3, depending on where the negative sign was placed.

$\frac{-2}{3}n \le 2$

$x\ 3 \quad \underline{x\ 3}$

In this case, the negative sign is assigned to the 2.

$\frac{-2n}{-2} \le \frac{6}{-2}$

Divide both sides of the inequality by -2. Since the number being moved is being DIVIDED by a negative number, be sure to reverse the inequality in the final answer.

n ≥ -3

If the negative sign is assigned to the 3 instead of the 2, the problem would be solved like this:

$\frac{2}{3}n \le 2$

$x\ 3 \quad \underline{x\ -3}$

Since the number being moved is being MULTIPLIED by a negative number, the inequality needs to be reversed in the final answer.

$\frac{2n}{2} \le \frac{-6}{2}$

n ≥ -3

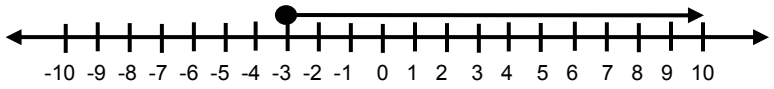
Ex 2: $-\frac{2}{3}n \leq 2$ Method 2: The other method that could be used to solve an inequality that contains a fraction is to multiply each side by the reciprocal. Don't forget to reverse the inequality in the final answer since the problem is being MULTIPLIED by a negative

~~3/2 ·~~ $-\frac{2}{3}n \leq 2 \cdot 3/2$

$n \leq -\frac{6}{2}$ or $n \leq -3$ Now reverse the sign in the final answer

n ≥ -3 Graph the final answer

$n \geq -3$



Practice Problems – Unit 3 – Section 3

For problems 1 -6 , solve and graph each inequality.

1) $\frac{r}{4} \geq -1$ 2) $1 \leq -\frac{w}{2}$ 3) $-\frac{v}{3} > 0.5$ 4) $-\frac{5}{8} < -\frac{5}{6}n$

5) $-1.5d < -6$ 6) $-7b > 42$

7) The science club charges $4.50 per car at their car wash. Write and solve an inequality to find how many cars they have to wash to earn at least $300.

8) Suppose you earn $6.15 per hour working part time at a dry cleaner. Write and solve an inequality to find how many full hours you must work to earn at least $100.

For questions 9 and 10, write four solutions to each inequality.

9) $-\frac{3}{4}q > 4$ 10) $-4.5 \leq -0.9p$

For questions 11 – 15, solve each inequality.

11) $-h \geq 4$ 12) $\frac{5}{2}x > 5$ 13) $-0.2m \geq 9.4$ 14) $\frac{3}{2}k \geq -45$

15) $4n \geq 9$

Unit 3 – Solving Inequalities

Section 4 – Solving Multi-Step Inequalities

(Refer to Page 207, problem numbers 37 - 66 in your textbook for additional practice)

Vocabulary and Examples:

To solve Multi-Step Inequalities - these problems are solved exactly like multi-step equations, the difference is that instead of an equal sign in the problem, there is an inequality AND most likely, the answer will be graphed on a number line.

IMPORTANT - if the number being moved is either multiplied or divided AND is NEGATIVE, the inequality in the final answer MUST BE REVERSED IN DIRECTION

Ex 1: Solve $7 + 6a > 19$

$7 + 6a > 19$ — Subtract 7 from both sides

$-7 \quad -7$

$\frac{6a}{6} > \frac{12}{6}$ — divide both sides by 6

$a > 2$

Ex 2: Solve $2(t + 2) - 3t \geq -1$

$2(t + 2) - 3t \geq -1$ — Use the distributive property

$2t + 4 - 3t \geq -1$ — Combine like terms

$-1t + 4 \geq -1$ — Subtract 4 from both sides

$-4 \quad -4$

$\frac{-1t}{-1} \geq \frac{-5}{-1}$ — Divide both sides by -1 AND reverse the inequality symbol

$t \leq 5$

Ex 3: Solve $6z - 15 < 4z + 11$

$6z - 15 < 4z + 11$ — Move the variables to the left side by subtracting 4z from both sides

$6z - 15 < 4z + 11$

$-4z \quad -4z$

$2z - 15 < 11$ — Add 15 to both sides

$+15 \quad +15$

$\frac{2z}{2} < \frac{26}{2}$ — Divide both sides by 2

$z < 13$

Ex 4: Solve $-3(4 - m) \geq 4(2m + 1)$

$-3(4 - m) \geq 4(2m + 1)$ — Use the distributive property on both sides

$-12 + 3m \geq 8m + 4$
$\underline{-8m \quad -8m}$ — Subtract 8m from both sides

$-12 - 5m \geq 4$
$\underline{+12 \qquad +12}$ — Add 12 to each side

$\frac{-5m}{-5} \geq \frac{16}{-5}$ — Divide both sides by -5 AND reverse the the inequality

$$\mathbf{m \leq -3\frac{1}{5}}$$

Practice Problems – Unit 3 – Section 4

For problems 1 -12 , solve each inequality.

1) $2(j - 4) \geq -6$

2) $-(6b - 2) > 0$

3) $3w + 2 < 2w + 5$

4) $2k + 6 \leq 4 + 5k$

5) $9 + x < 7 - 2(x - 3)$

6) $25 > -(4y + 7)$

7) $\frac{4}{3}r - 3 < r + \frac{2}{3} - \frac{1}{3}r$

8) $5x - \frac{1}{2}(3x + 8) \leq -4 + 3x$

9) $5c + 4(c - 1) \geq 2 + 5(2 + c)$

10) $4(3m - 1) \geq 2(m + 3)$

11) $3(3r + 1) - (r + 4) \leq 13$

12) $3.8 - k \leq 5.2 - 2k$

13) On a trip from Virginia to Florida, the Sampson family wants to travel at least 420 miles in 8 hours of driving. What must be their average rate of speed?

14) The school band needs a banner to carry in a parade. The banner committee decides that the length of the banner should be 18 feet. A committee member drew the diagram below to help understand the problem. What are the possible widths of the banner if they can use no more than 48 feet of trim? (HINT since the border goes around the edges of a rectangular banner, use the perimeter formula P = 2l + 2w)

15) Joleen is a sales associate in a clothing store. Each week she earns $250 plus a commission equal to 3% of her sales. This week her goal is to earn no less than $460. Write and solve an inequality to find the dollar amount of the sales she must have to reach her goal. (HINT: remember to change the percent to its decimal equivalent in the inequality)

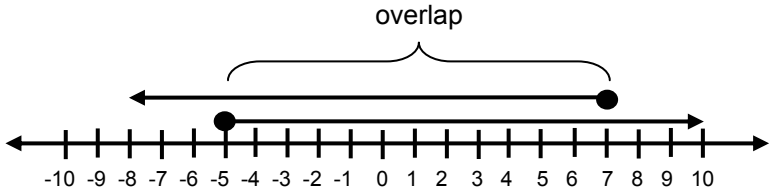
Unit 3 – Solving Inequalities
Section 5 – Compound Inequalities

Vocabulary and Examples:

Compound Inequality – two inequalities that are joined by the word "*and*" or the word "*or*"

Ex: $x \geq -5$ and $x \leq 7$

If these two inequalities were graphed on the same number line, there would be an overlap of the solutions. In addition to the overlap, notice that the numbers -5 and 7 become the border numbers of the final solution on the graph



Because of this overlap, the 2 original inequalities can be rewritten as one compound inequality:

$-5 \leq x \leq 7$ This is read as: x is greater than or equal to -5 and less than or equal to 7

Notice: (1) the border numbers are written from least to greatest
(2) if the numbers are written from least to greatest, the inequality symbols will ALWAYS be $\leq$ or $<$ (Never $>$ or $\geq$)
(3) the x represents the overlapped numbers on the number line which is the solution to the inequality
(4) anytime there is overlap when the solution is graphed, the compound inequality will be written with the variable in the middle and is considered an "and" problem

Ex 1: write an inequality to represent: all real numbers that are at least -2 and at most 4

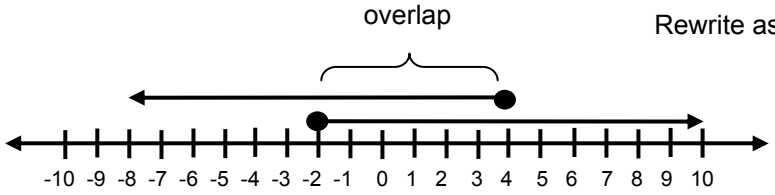
To determine the inequality, make sure to represent what the problem is asking

1) the numbers that are "at least" -2 are those numbers that are -2 or more since -2 is the least the number can be.

2) the numbers that are ""at most" 4, are those numbers that are less than or equal to 4 since the 4 is the most the number can be

3) pick a letter to represent all of the number that fall between -2 and 4

$n \geq -2$ and $n \leq 4$

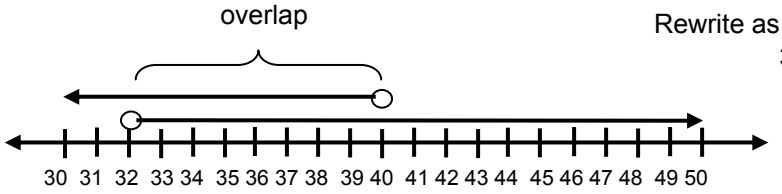


Rewrite as a compound inequality:
$-2 \leq n \leq 4$

Ex 2: Today's temperatures will be above 32° F, but not as high as 40° F

above 32° F is "more than" 32 $t > 32$

not as high as 40° F is "less than" 40 $t < 40$



Rewrite as a compound inequality:

$32 < n < 40$

Ex 3: solve $-4 < r - 5 \leq -1$

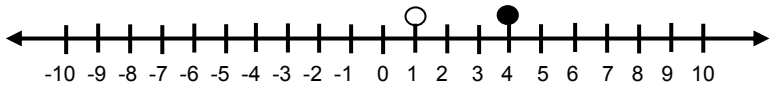
isolate the variable in the middle by moving the -5 out of the middle to both sides by adding 5 to the -4 and the -1

$$\begin{array}{rrrr} -4 & < r - \cancel{5} & \leq & -1 \\ +5 & \cancel{+5} & & +5 \end{array}$$

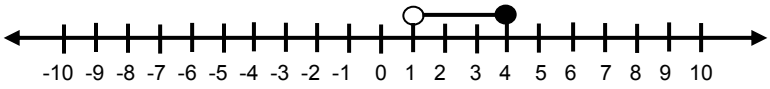
$1 < r \leq 4$

Notice once the variable is isolated, what is left is a compound inequality

To graph the answer, place a open circle above the 1 on a number line and a closed circle above the 4 on the same number line



Then draw a line connecting the two circles



Any time the variable is in the middle, the two circles will always be connected with a line. Make sure to pay attention to the inequality symbol to determine whether the circles are open or closed

Ex 4: The acidity of the water in a swimming pool is considered normal if the average of the three pH readings is between 7.2 and 7.8, inclusive (including 7.2 and 7.8). The first two readings for a swimming pool are 7.4 and 7.9. What possible values for the third reading p will make the average pH normal?

Notice: 1) The problem refers to average (adding all of the data together and dividing by the number of data (in this case 3 because of the three readings needing for the pH of the pool water)

$$\frac{7.4 + 7.9 + x}{3} \longrightarrow \text{The third unknown reading}$$

2) the average of the three pH readings is BEWTEEN 7.2 and 7.8, including 7.2 and 7.8

$$7.2 \leq \frac{7.4 + 7.9 + x}{3} \leq 7.8$$

notice the numbers are listed from smallest to largest . Because of this, the inequalities must be either $\leq$ or $<$. In this case, because the problem specifically states "inclusive", the inequality is $\leq$

3) solve for x by isolating the variable in the middle

$$7.2 \leq \frac{7.4 + 7.9 + x}{3} \leq 7.8$$

$$\underline{\times 3} \qquad \cancel{\times 3} \qquad \underline{\times 3}$$

multiply all sides by 3

$$21.6 \leq 7.4 + 7.9 + x \leq 7.8$$

combine like terms

$$21.6 \leq 15.3 + x \leq 7.8$$

$$\underline{-15.3} \quad \cancel{-15.3} \quad \underline{-15.3}$$

subtract 15.3 from all sides

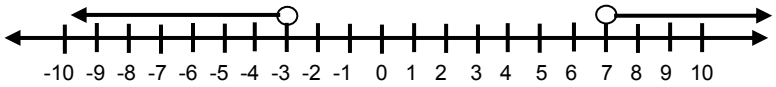
$6.3 \leq x \leq 8.1$

The value for the third reading must be between 6.3 and 8.1, inclusive

Ex 5: write a compound inequality that represents:
all real numbers that are less than -3 or greater than 7

1) Rewrite as a compound inequality $c < -3$ OR $c > 7$

2) Graph



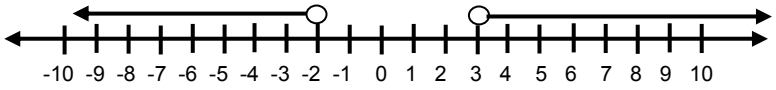
3) Notice: when the original problem includes the word "or", there is no overlap when graphed. In fact, the arrows will ALWAYS point in opposite directions.

Ex 6: solve the compound inequality: $4v + 3 < -5$ or $-2v + 7 < 1$

1) Solve each inequality separately. Notice the word "or" - when graphed, the arrows will point in opposite directions

Subtract 3 from both sides	$4v + 3 < -5$ $-3 \quad -3$	$-2v + 7 < 1$ $-7 \quad -7$	Subtract 7 from both sides
Divide both sides by 4	$\frac{4v}{4} < \frac{-8}{4}$	$\frac{-2v}{-2} < \frac{-6}{-2}$	Divide both sides by -2
	$\mathbf{v < -2}$	$\mathbf{v > 3}$	Flip the inequality

2) Graph the solutions on the same number line



Practice Problems – Unit 3 – Section 5

For problems 1 -2 , write a compound inequality that represents the situation.

1) all real numbers that are between -4 and 6

2) all the real numbers that are at most -5 or at least 3

For problems 3 – 12, solve each compound inequality and graph your solution.

3) $-3 < j + 2 < 7$

4) $-1 < 4m + 7 \leq 11$

5) $3 \geq 4r - 5 \geq -1$

6) $12 \leq \frac{14 + 17 + a}{3} \leq 16$

7) $\frac{1}{2} < \frac{3x - 1}{4} < 5$

8) $3.2 \geq 2r + 0.2 > -3.8$

9) $3b - 1 < -7$ or $4b + 1 > 9$

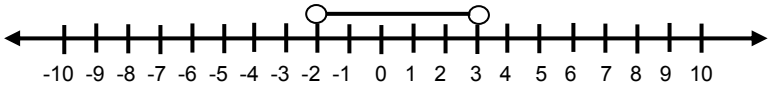
10) $2d + 5 \leq -1$ or $-2d + 5 \leq 5$

11) $5y + 7 \leq -3$ or $3y - 2 \geq 13$

12) $3c + 4 \geq 13$ or $6c - 1 < 11$

For problems 13 – 15, write a compound inequality that each graph could represent.

13)



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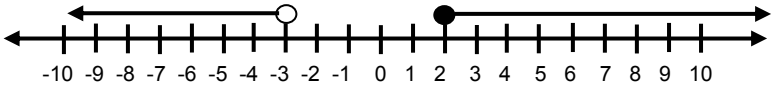

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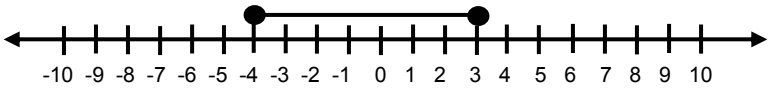


Unit 3 – Solving Inequalities

Section 6 – Absolute Value Equations and Inequalities

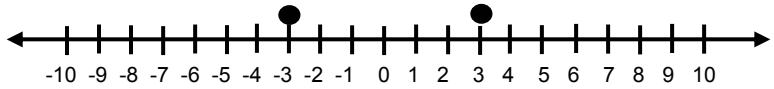
(Refer to Page 197, problem numbers 1 - 20 in your textbook for additional practice)

Vocabulary and Examples:

Absolute value – remember that the absolute value of a number is its distance from zero on a number line

Ex: $|x| = 3$ x = -3 or 3 since both 3 and -3 are 3 units from zero on a number line

The graph of $|x| = 3$ is:



Ex 1: solve $|x| + 5 = 11$

first, move any number not INSIDE the absolute value symbols to the other side of the equal sign. In this case, subtract 5 from both sides.

$|x| + \cancel{5} = 11$
$\cancel{-5} \quad -5$

$|x| = 6$

x = 6 or x = -6

Ex 2: solve $|2p + 5| = 11$

first, move any number not INSIDE the absolute value symbols to the other side of the equal sign. In this case, there is no number to move

second, remove the absolute value symbols and write two equations – one equation is the original without the absolute value symbols, and the second equation is the original with the answer changed to its opposite.

$2p + 5 = 11$ ← write two equations → $2p + 5 = -11$

$2p + \cancel{5} = 11$ ← subtract 5 from each side → $2p + \cancel{5} = -11$
$\cancel{-5} \quad -5$ $\qquad\qquad$ $\cancel{-5} \quad -5$

$\frac{\cancel{2}p}{\cancel{2}} = \frac{6}{2}$ ← divide each side by 2 → $\frac{\cancel{2}p}{\cancel{2}} = \frac{-16}{2}$

p = 3 $\qquad\qquad$ **p = -8**

the value for p is 3 or -8

Ex 3: solve and graph $|n - 1| < 3$

first, move any number not INSIDE the absolute value symbols to the other side of the equal sign. In this case, there is no number to move

second, remove the absolute value symbols and write two inequalities – one inequality is the original without the absolute value symbols, and the second inequality is the original with the answer AND the inequality changed to its opposite.

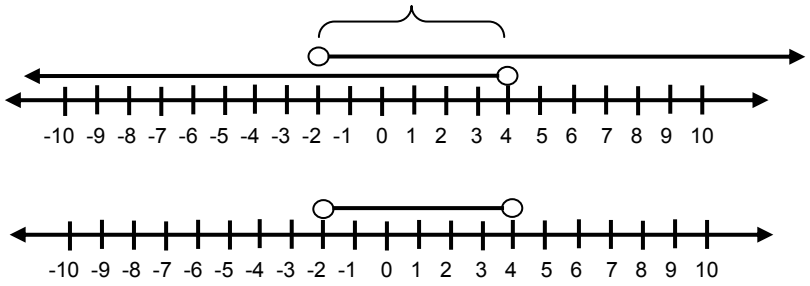
$n - 1 < 3$ write as a compound inequality $n - 1 > -3$

$n - 1 < 3$ ← add 1 to each side → $n - 1 > -3$

$+1 \quad +1$ $+1 \quad +1$

$n < 4$ **$n > -2$**

overlap



$-2 < n < 4$

Notice that when the ORIGINAL problem has an inequality of "is less than" or "is less than or equal to", the graphed solution shows overlap, and therefore is an "and" problem. The compound inequality solution will be written with the variable in the middle and the two "border" numbers written from least to greatest. The inequality will be either "$<$" or "$\leq$" depending on what was given in the original problem.

Ex 4: solve and graph $|v - 3| \geq 4$

first, move any number not INSIDE the absolute value symbols to the other side of the equal sign. In this case, there is no number to move

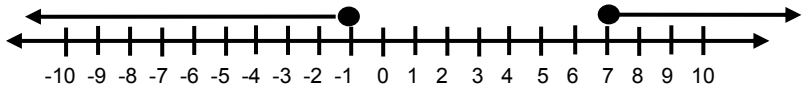
second, remove the absolute value symbols and write two inequalities – one inequality is the original without the absolute value symbols, and the second inequality is the original with the answer AND the inequality changed to its opposite.

$v - 3 \geq 4$ write as a compound inequality $v - 3 \leq -4$

$v - 3 \geq 4$ ← add 3 to each side → $v - 3 \leq -4$

$+3 \quad +3$ $+3 \quad +3$

$v \geq 7$ **$n \leq -1$**



$n \leq -1$ or $n \geq 7$

Notice that when the ORIGINAL problem has an inequality of "is greater than" or "is greater than or equal to", the graphed solution shows arrows going in opposite directions, and therefore is an "or" problem. The compound inequality solution will be written with an "or" between the two solutions

Ex 5: solve $-6|w - 3| < -24$

notice there is a -6 outside the absolute value symbols. This -6 needs to be distributed through the terms inside the absolute value symbols before anything else can be done. As the distributive property is being done, the absolute value symbols are removed.

$-6w + 18 < -24$

$-6w + 18 < -24$ write as a compound inequality $-6w + 18 > 24$

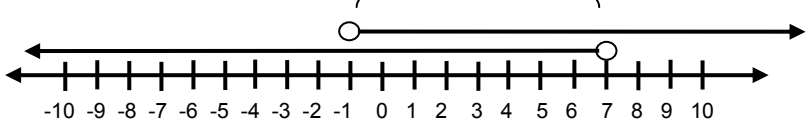
$-6w + 18 < -24$ ← subtract 18 from each side → $-6w + 18 > 24$

$-18 \quad -18$ $-18 \quad -18$

$\frac{-6w}{-6} < \frac{-42}{-6}$ ← divide each side by -6 → $\frac{-6w}{-6} > \frac{6}{-6}$

overlap

$w < 7$ **$w > -1$**



$-1 < w < 7$

Because the original problem includes the inequality "is less than", the graphed solution shows overlap, and therefore the compound inequality will be written with the variable in the middle, and is an "and" problem

Practice Problems – Unit 3 – Section 6

For problems 1 -5 , solve each equation

1) $4 = |y|$

2) $|n| + 2 = 8$

3) $-3|m| = -6$

4) $16 = 5|p| - 4$

5) $-2|7d| = -14$

For problems 6 -8 , solve and graph each inequality.

6) $|w| < 2$

7) $|x + 3| < 5$

8) $|5t - 4| \geq 16$

For problems 9 - 15, solve each equation or inequality.

9) $|2d| + 3 = 21$

10) $|p| - \frac{2}{3} = \frac{5}{6}$

11) $-2|c - 4| = -8$

12) $\frac{|v|}{-3} = -4.2$

13) $\left|\frac{1}{2}a\right| + 1 = 5$

14) $|6.5x| < 39$

15) $4 - 3|m + 2| > -14$