Unit 4 – Solving and Applying Proportions

Section 1 – Ratio and Proportion

Vocabulary:

Ratio – a comparison of two numbers by division.

The ratio of *a* to *b* is $\frac{a}{b}$ $a:b$ or a:b, where b ≠ 0 $b \neq 0$.

Rate – a ratio where the two numbers represent quantities measured in different units.

Unit Rate – a rate with a denominator of 1. A common example would be the speed of an automobile, expressed as 65 miles/1 hour. This rate can be written as sixty-five miles per hour or 65 mi/hr.

Proportion – an equation stating two ratios are equal.

$\frac{a}{b} = \frac{c}{d}$., where b ≠0 and d ≠ 0. This proportion is read *a* is to *b* as *c* is to *d*.

Extremes of the proportion – *a* and *d* are the extremes of the proportion.

Means of the proportion – *b* and *c* are the means of the proportion.

Using Unit Rates

Cost of Soda		
Vending Machine	$1.00	16 oz
Grocery Store	$1.79	68 oz.

The table above gives prices for different sizes of the same brand of soda. Find the unit rate (cost per ounce) for the 16 oz. size.

$\frac{\$1.00}{16oz.}$ The cost per 16 ounces is expressed as a ratio.

To determine the unit rate, **divide the numerator ($1.00) by the denominator (16 oz.).**

The unit rate is $\frac{\$.0625}{1oz.}$

Exercise: Determine the unit rate for the 68 ounce bottle. Divide the numerator and denominator by 68 oz. to get the cost per 1 ounce,

Using Unit Rates (continued):

Unit rates can be converted from one unit of measure to another by multiplying them by rates equal to 1.

a) Continuing with the cost of soda, we can convert the cost of buying soda from a cost per ounce to a cost per gallon. 1 gallon equals 128 ounces, therefore a rate of $\frac{128 \text{ oz.}}{1 \text{ gal}}$ is considered a rate of 1.

Multiplying the per ounce cost of the 16 ounce bottle of soda by a rate of one would let us compare its price to something else we might purchase like a gallon of gasoline.

Unit 4 – Solving and Applying Proportions

Section 5 – Applying Ratios to Probability

Vocabulary:

Probability – Or P(event), Tells you how likely it is that something will happen.

Outcome – Is the result of a single trial, such as the flipping of a coin.

Event – An outcome or group of outcomes.

Sample space – All of the possible outcomes.

Theoretical Probability – when all possible outcomes are equally likely, the theoretical probability of an event is a ratio of the number of possible favorable outcomes divided by the number of possible outcomes.

Compliment of an event – all outcomes not in an event.

Experimental Probability – a ratio of the number of times an event occurs divided by the number of times the experiment is done.

OBJECTIVE #1: THEORETICAL PROBABILITY

The theoretical probability of an event is a ratio of the number of possible favorable outcomes divided by the number of possible outcomes. You can write the probability of an even as a fraction, a decimal, or percent. The probability of an event ranges from 0 to 1. A probability of 0 would be the probability of an impossible event. A probability of 1 would be a the probability of an event that is certain to happen.

We'll use a 6-sided number cube, numbered 1 through 6, to explain these terms.

Event	Sample Space	Favorable Outcome
↓	↓	↓
Rolling an even number	1,2,3,4,5,6	2,4,6

The possible outcomes of rolling a fair number cube are equally likely to occur. When all possible outcomes are equally likely to occur, you can find the theoretical probability using the following:

Theoretical Probability $P(event) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$

$$P(\text{rolling an even number}) = \frac{3}{6} = \frac{1}{2}$$

Where 0 probability is an impossible event, like rolling a 7 on a 6-sided number cube, and a probability of 1 is a certainty, like rolling a number less than 7 on a number cube, then a probability of $\frac{1}{2}$ says an event is equally likely and unlikely to occur, like getting a heads when flipping a coin.

EXAMPLE: FINDING THEORETICAL PROBABILITY
A bowl contains twelve slips of paper, each with a different name of the month. Find the theoretical probability that a slip of paper selected at random from the bowl has a name of a month that starts with the letter J.

$$P\text{ (event)} = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

$= \frac{3}{12}$ There are three out of twelve months that begin with the letter J.

$= \frac{1}{4}$ Simplify

OBJECTIVE #1 (cont):

The ***compliment of an event*** consists of all outcomes not in the event. In the previous example of rolling a 6-sided number cube where we were looking for the probability of rolling an even number, the compliment of that event would be rolling an odd number.

possible outcomes for rolling a number cube	outcomes for rolling an even number	compliment of rolling an even number
↓	↓	↓
1,2,3,4,5,6	2,4,6	1,3,5

The sum of the probability of an event and its compliment is 1.

P(event) + P(compliment of the event) = 1 or, written another way,

P(compliment of the event) = 1 – P(event)

EXAMPLE: Finding the Compliment of an Event
On a popular game show, contestants pick a suitcase containing money. If there were 10 suitcases, and only one contained the grand prize of $1 million, what is the probability of not picking the suitcase containing the grand prize.

$$P(\text{winning \$1 million}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{1}{10}$$

P(not winning $1 million) = 1 – P(winning $1 million)

$$= 1 - \frac{1}{10} = \frac{9}{10}$$

OBJECTIVE #2: EXPERIMENTAL PROBABILITY

Probability based on data collected from repeated trials is *experimental probability*. Experimental probability is often used to predict outcomes.

You can find experimental probability using the following formula:

$$\text{Experimental Probability} = \frac{\text{number of times an event occurs}}{\text{number of times the experiment is done}}$$

EXAMPLE: Using Experimental Probability

In a recent outbreak of an eye fungus related to wearing contact lenses, 50 of 109 people who contracted the fungus were interviewed. Of the 50 interviewed, 46 had used a particular lens care brand and the other four had used a brand manufactured by the same company. Based on this very high experimental probability, the manufacturer's products were pulled off the market.

Quality Control uses Experimental Probability:
Inspectors test 500 electronic computer chips off a production line. Their test results show 494 of the 500 with no defects.

$$P(\text{no defects}) = \frac{\text{number of times an event occurs}}{\text{number of times an experiment is done}}$$

$$= \frac{494}{500} \quad \text{Substitute}$$

$$= 0.988 \quad \text{Simplify}$$

$$= 98.8\% \quad \text{Write as a percent}$$

The probability that a chip has no defects is 99.8%.

In a production run of 500,000, this would mean they will produce 494,000 chips with no defects. The compliment of this event would be that they will produce 6,000 defective chips.

Unit 4 Section 5 worksheet

For Exercises 1-12, you have six tiles numbered 1 through 6. Two of the tiles are red, three of the tiles are purple, and one of the tiles is green. Find the theoretical probability of the following events:

1. P(red)
2. P(purple)
3. P(2)
4. P(red or green)
5. P(1 or 6)
6. P(white)
7. P(even)
8. P(not red)
9. P(7)
10. P(even or odd)
11. P(not green)
12. P(less than 3)

Suppose you roll a 6-sided number cube. Find each probability.

13. P(5)
14. P(8)
15. P(1 or 2)
16. P(not 6)
17. P(less than 5)
18. P(4 or higher)

Suppose you select a 3-digit number at random from the set of all positive 3-digit numbers. Find each probability. (Hint: First find how many positive 3-digit numbers there are.)

19. P(odd number)
20. P(141 or 144)
21. P(number less than 900)
22. P(number less than 100)
23. P(even number)
24. P(number is a multiple of 30)
25. P(number less than 500)
26. P(888)

Unit 4 SECTION 5 WORKSHEET – ANSWER SHEET

For Exercises 1-12, you have six tiles numbered 1 through 6. Two of the tiles are red, three of the tiles are purple, and one of the tiles is green. Find the theoretical probability of the following events:

1. P(red) **2/6**
2. P(purple) **1/2**
3. P(2) **1/6**
4. P(red or green) **1/2**
5. P(1 or 6) **1/3**
6. P(white) **0**
7. P(even) **1/2**
8. P(not red) **2/3**
9. P(7) **0**
10. P(even or odd) 1
11. P(not green) **5/6**
12. P(less than 3) **1/3**

Suppose you roll a 6-sided number cube. Find each probability.

13. P(5) **1/6**
14. P(8) **0**
15. P(1 or 2) **1/3**
16. P(not 6) **5/6**
17. P(less than 5) **2/3**
18. P(4 or higher)**1/2**

Suppose you select a 3-digit number at random from the set of all positive 3-digit numbers. Find each probability. (Hint: First find how many positive 3-digit numbers there are.)

19. P(odd number) **1/2**
20. P(141 or 144) **1/450**
21. P(number less than 900) **8/9**
22. P(number less than 100) **0**
23. P(even number)**1/2**
24. P(number is a multiple of 30)**1/30**
25. P(number less than 500)**4/9**
26. P(888) **1/900**

Unit 4 – Solving and Applying Proportions
Section 4 – Percent of Change

Vocabulary:

Percent of Change – a ratio comparing the amount of change to the original amount expressed as a percent $\frac{\text{amount of change}}{\text{original amount}}$.

Percent of Increase – the Percent of Change measuring the increase of an amount from its original amount.

Percent of Decrease – the Percent of Change measuring the decrease of an amount from its original amount.

Greatest Possible Error – the greatest possible error in a measurement is one half of the unit of measure.

OBJECTIVE #1 – PERCENT OF CHANGE

The percent of change compares the increase or decrease of an amount as compared to the original amount.

Step 1. Find the absolute value of the change in amount.

Step 2. Divide the change in amount by the original amount and express as a percent.

Example 1: The price of a bicycle decreased from $150.00 to $126.00. Find the percent of decrease.

$$\text{Percent of Decrease} = \frac{\text{Amount of Change}}{\text{Original Amount}}$$

$$= \frac{150.00 - 126.00}{150.00}$$

$$= \frac{24.00}{150.00}$$

= 0.16 or 16%

The price of the bicycle decreased by 16%.

`

Example 2: The price of a gallon of gasoline at the Bush's Oil R'Us gas station went from $2.04 per gallon to $2.57per gallon. About how much was the percent of increase?

$$\text{Percent of Decrease} = \frac{\text{Amount of Change}}{\text{Original Amount}}$$

$$= \frac{\$2.57 - \$2.04}{\$2.04}$$

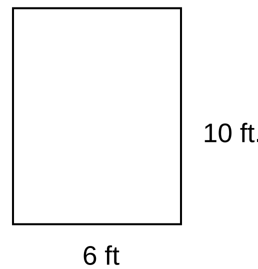
$$= \frac{\$.53}{\$2.04}$$

= approx. 26% Increase

OBJECTIVE #2: Greatest Possible Error

The <u>greatest possible error</u> in a measurement is one half of that measuring unit; for example, if you were measuring a picture for a picture frame and measured it as 6.1 inches, then the <u>greatest possible error</u> would be .05 inches. The measuring unit was tenths of inches, so the greatest possible error would be half of one-tenth of an inch, or 0.05 inches

Example #1: You measure a room and make a diagram shown at the right with the room measurements. Use the greatest possible error to find the minimum and maximum possible areas.



Both measurements were made to the nearest whole foot, so the greatest possible error is 0.5 ft. The length could be as little as 9.5 ft. or as great as 10.5 ft. The width could be as little as 5.5 ft. or as great as 6.5 ft.

Minimum Area	**Maximum Area**
9.5 ft. x 5.5 ft. = 52.25 sq. ft.	10.5 ft. x 6.5 ft. = 68.25 sq. ft.

The minimum area is 52.25 sq. ft. and the maximum area is 68.25 sq. ft.

<u>Percent error</u> is another useful way to think of the error in a measurement. It is the error of the greatest possible error and the measurement.

$$\text{Percent Error} = \frac{\text{greatest possible error}}{\text{measurement}}$$

Example #2: Suppose you measure a CD and record it's measurement as 12.1 cm. Find the percent error in your measurement.

Since the measurement is to the nearest 0.1 cm., the greatest possible error is 0.05 cm.

$$\text{Percent Error} = \frac{\text{greatest possible error}}{\text{measurement}}$$ Use the percent error formula.

$$= \frac{0.05}{12.1}$$ Substitute

= 0.0041322 Divide

= 0.4 % Round and write as percent.

Example #3: Finding the percent error in calculating volume.

Suppose you measure a card box and record the following dimensions; 7.9 cm x 4.7 cm. x 1.2 cm.
As measured the volume of the box = 7.9 cm. x 4.7 cm. x 1.0 cm. = 37.13 cubic cm.

-Using maximum values, the volume = 7.95 cm. x 4.75 cm. x 1.05 cm. = 39. 65 cubic cm. (rounded)
-Using minimum values, the volume = 7.85 cm. x 4.65 cm. x 0.95 cm. = 34.68 cubic cm.

Possible Error:

Using **maximum** minus **measured =** 39.65 – 37.13 = 2.52
Using **measured** minus **minimum** = 37.13 – 34.68 = 2.45

Use the larger of the differences to calculate the percent error in calculating the volume.

$$\text{Percent Error} = \frac{\text{greatest possible error}}{\text{measurement}}$$ Use the percent error formula.

$$= \frac{2.52}{37.13}$$ Substitute

$= 0.0678696$ Divide

$= 6.8\ \%$ Round and write as a percent.

The percent error is about 6.8%

Unit 4 SECTION 4 WORKSHEET

Find each percent of change. Describe each percent of change as an increase or a decrease. Round to the nearest tenth if necessary.

1. $4 to $5
2. $5 to $4
3. 2 ft. to 3 ft.
4. 3 ft. to 2 ft.
5. 10 m to 12 m
6. 12 m to 9 m
7. 4.5 in to 9 in
8. 9 in to 4.5 in
9. $28.40 to $32.00
10. $32.00 to $28.40
11. 180 lbs to 220 lbs.
12. 220 lbs to 180 lbs

Find the greatest possible error for each measurement:

13. 45 ft.
14. 87.3 gms
15. 96 lbs.
16. 13 m
17. 16 tons
18. 4 miles

Find the minimum and maximum possible areas for rectangles with the following measurements:

19. 7 km x 8 km
20. 12 mi. x 5 mi.
21. 4 in. x 6 in.
22. 18 cm x 15 cm.
23. 23 in x 14 in
24. 6 km x 9 km

Find the percent error of each of the following measurements:

25. 4 cm.
26. 0.4 cm.
27. 2 cm.
28. 0.2 cm.

You have a rectangular prism that has a recorded measurement of 8 cm. x 3 cm. x 2 cm. Calculate the following:

a) the measured volume
b) the maximum volume
c) the minimum volume
d) the greatest possible error
e) the percent error (rounded to the nearest percent)

Unit 4 SECTION 4 WORKSHEET – ANSWER SHEET

Find each percent of change. Describe each percent of change as an increase or a decrease. Round to the nearest tenth if necessary.

1. $4 to $5 25% I
2. $5 to $4 20% D
3. 2 ft. to 3 ft. 50% I
4. 3 ft. to 2 ft. 33.3% D
5. 10 m to 12 m 20% I
6. 12 m to 9 m 25% D
7. 4.5 in to 9 in 100% I
8. 9 in to 4.5 in 50% D
9. $28.40 to $32.00 12.7%I
10. $32.00 to $28.40 11% D
11. 180 lbs to 220 lbs.22.2%I
12. 220 lbs to 180 lbs18.2%D

Find the greatest possible error for each measurement:

13. 45 ft. 0.5 ft.
14. 87.3 gms 0.05 gms
15. 96 lbs. 0.5 lbs
16. 13 m 0.5 m
17. 16.4 cm 0.05 cm
18. 4.2 mi. 0.05 mi.

Find the minimum and maximum possible areas for rectangles with the following measurements:

19. 7 km x 8 km 48.75 km^2/63.75 km^2
20. 12 mi. x 5 mi.51.75mi^2/68.75 mi^2
21. 4 in. x 6 in. 19.25in^2/29.25 in^2
22. 18 cm x 15 cm.253.75cm^2/286.75cm^2
23. 23 in x 14 in 303.75 in^2/340.75 in^2
24. 6 km x 9 km 46.75km^2/61.75 km^2

Find the percent error of each of the following measurements:

25. 4 cm.12.5%
26. 0.4 cm.12.5%
27. 2 cm.25%
28. 0.2 cm.25%

You have a rectangular prism that has a recorded measurement of 8 cm. x 3 cm. x 2 cm. Calculate the following:

f) the measured volume 48 cubic cm.
g) the maximum volume 74.375 cubic cm.
h) the minimum volume 28.125 cubic cm
i) the greatest possible error 26.375 cubic cm
j) the percent error (rounded to the nearest percent) 55%

Unit 4 – Solving and Applying Proportions
Section 3 – Proportions and Percent Equations

Vocabulary:

Percent – a ratio that compares a number to 100.

OBJECTIVE #1: APPLYING PROPORTIONS TO PERCENT PROBLEMS.

Percent problems can be solved by writing and solving a proportion, where $\frac{n}{100} = \frac{part}{whole}$ or, expressed another way, $\frac{n}{100} = \frac{is}{of}$. "n" is the percent number.

Example #1 – Finding the Percent

What percent of 60 is 18?

Set up the proportion: $\frac{n}{100} = \frac{part}{whole}$ The "of" number represents the whole and in this case it is 60. The "part" number, or the "is" number in this problem is 18. Therefore, $\frac{n}{100} = \frac{18}{60}$

$60n=1800$ Find the cross products.
$n=30.0$ Divide each side by 60.

30% of 60 is 18.

Example #2 – Finding the Part

Find 45% of 240.

$\frac{n}{100} = \frac{part}{whole}$ therefore, we set up the proportion $\frac{45}{100} = \frac{n}{240}$, where 45 is the percent number. The "whole" or "of" number is 240. We are looking for the "is" number.

$\frac{45}{100} = \frac{n}{240}$ Set up the proportion.
24,500=100n Find the cross products.
245=n Divide each side by 100.

Example #3 – Finding the Whole

18 is 60% of what number?

$\frac{n}{100} = \frac{part}{whole}$, therefore we set up the proportion $\frac{60}{100} = \frac{18}{n}$ where 60 is the percent number, and 18 is the "part", or "is", number. We are looking for the whole number.

$\frac{60}{100} = \frac{18}{n}$ Set up the proportion.
60n = 1800 Find the cross products.
n = 30 Divide both sides by 60.

OBJECTIVE #2: SOLVING PERCENT PROBLEMS BY TRANSLATING WORDS INTO EQUATIONS.

Percent problems can be solved by translating the words into an equation.

Example #4 – Using a Percent Equation

What percent of 180 is 54?

Relate What percent of 180 is 54?

Define Let p = the decimal form of percent.

Write $p \cdot 180 = 54$

$180p = 54$
$p = 0.3$ Divide each side by 180.
$p = 30\%$ Write the decimal as a percent.

Remember that percents can be greater than 100% and less than 1%.

Example #5 – What percent of 120 is 168?

$n \cdot 120 = 168$

$120n = 168$ Write an equation.

$n = 1.4$ Divide each side by 120.

$n = 140\%$ Write the decimal as a percent.

140% of 120 is 168.

Example #6 – What is 0.65% of 200?

$n = 0.0065 \cdot 200$
$= 1.3$

1.3 is 0.65% of 200.

Solve the problems on the Unit 4 Section 3 Worksheet.

Unit 4 SECTION 3 WORKSHEET

Solve each problem using a proportion.

1. What percent of 50 is 25?
2. What percent of 80 is 20?
3. 12 is what percent of 48?
4. What percent of 60 is 18?
5. 36 is what percent of 90?
6. 30% of 70 is what number?
7. 45 is what percent of 180?
8. 8% of 125 is what number?
9. What percent of 20 is 19?
10. What percent of 20 is 125?

Write a proportion and solve to find the whole.

11. 24 is 30% of what number?
12. 42 is 25% of what number?
13. 120% of what number is 72?
14. 60% of what number is 75?
15. 20% of what number is 48?
16. 26 is 20% of what number?
17. Amy saved $120 towards new MP3 player she wants to buy. This represents 60% of the cost of the MP3 player. How much does the MP3 player cost?

Solve each problem.

18. 3 is 75% of what number?
19. What percent of 75 is 300?
20. What is 0.3% of 800?
21. 1.8 is 4% of what number?
22. 160% of 90 is what number?
23. 965 is what percent of 1000?
24. John works in an appliance store and earns an 8% commission on everything he sells. How much does he earn if he sells a $1200 refrigerator?
25. Allison plans to buy a car costing $22, 500. The sales tax where she lives is 6%. How much additional will the car cost in sales tax?

Unit 4 SECTION 3 WORKSHEET – ANSWER SHEET

Solve each problem using a proportion.

1. What percent of 50 is 25? 50%
2. What percent of 80 is 20? 25%
3. 12 is what percent of 48? 25%
4. What percent of 60 is 18? 30%
5. 36 is what percent of 90? 40%
6. 30% of 70 is what number? 21
7. 45 is what percent of 180? 25%
8. 8% of 125 is what number? 10
9. What percent of 20 is 19? 95%
10. What percent of 20 is 125? 625%

Write a proportion and solve to find the whole.

11. 24 is 30% of what number? 80
12. 42 is 25% of what number? 168
13. 120% of what number is 72? 60
14. 60% of what number is 75? 125
15. 20% of what number is 48? 240
16. 26 is 20% of what number? 130
17.
18. Amy saved $120 towards new MP3 player she wants to buy. This represents 60% of the cost of the MP3 player. How much does the MP3 player cost? $200

Solve each problem.

19. 3 is 75% of what number? 4
20. What percent of 75 is 300? 400%
21. What is 0.3% of 800? 240
22. 1.8 is 4% of what number? 45
23. 160% of 90 is what number? 144
24. 965 is what percent of 1000? 96.5%
25. John works in an appliance store and earns an 8% commission on everything he sells. How much does he earn if he sells a $1200 refrigerator? $96

26. Allison plans to buy a car costing $22, 500. The sales tax where she lives is 6%. How much additional will the car cost in sales tax? $1350

Unit 4 – Solving and Applying Proportions
Section 2 – Proportions and Similar Figures

Vocabulary:

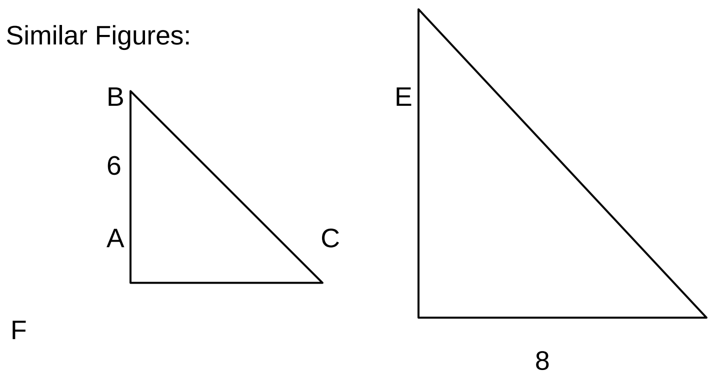
Similar figures – have the same shape but not necessarily the same size.
The symbol “ ~ ” means “is similar to”.

Scale drawing – an enlarged or reduced drawing that is similar to and actual object or place. Examples of scale drawings would be the blueprints for building a house or a map.

Scale – gives the ratio of the distance in the drawing compared to the corresponding actual distance.

Congruent – means “the same as” when referring to shapes, angles, or length of sides of shapes. The symbol “ $\cong$ ” means “is congruent to” or “the same as”.

Applying Proportions to Similar Figures:



Triangle ABC ~ Triangle DEF, or written another way, $\triangle ABC \sim \triangle DEF$.
Triangle ABC is similar to Triangle DEF. The corresponding angles are congruent and the corresponding sides are in proportion to each other; therefore,

$$\frac{\text{AB}}{\text{AC}} = \frac{\text{DE}}{\text{DF}} \text{ OR } \frac{6}{5} = \frac{DE}{8}.$$

By solving the proportion for DE, you can determine the length of side DE.

$$(6)(8) = (5)\bullet\text{DE} \;\therefore\; \text{DE}=9.6$$

Applying Proportions to Similar Figures (continued)

Indirect Measurement – Proportions can be used to find the dimensions of objects that are difficult to measure.

Example: A flag pole casts a shadow 15 ft. long. A man 6 ft. tall casts a shadow that is 4 ft. long. The triangle made by the flag pole and its shadow is similar to the triangle made by the man and his shadow; therefore, we can set up the following proportion:

$$\frac{\text{Height of the flag pole}}{\text{Height of the man}} = \frac{\text{Length of flag pole shadow}}{\text{Length of man's shadow}} \text{ or } \frac{\text{H}}{6'} = \frac{15'}{4'}$$

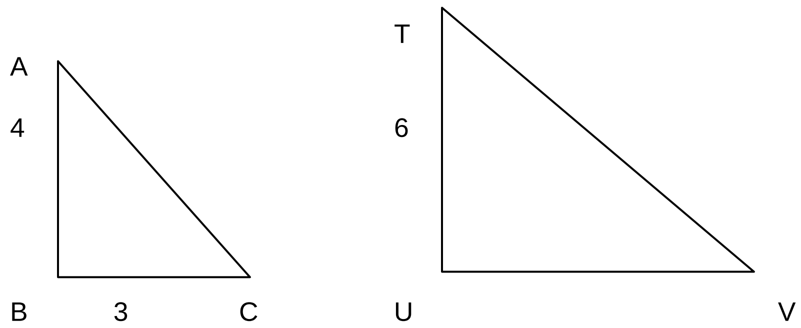
$$\text{H}= \frac{(6)\bullet(15)}{4} = 22.5'$$ The flag pole is 22.5’ tall.

Distances on Maps – all maps include a scale which relates a measurable distance on the map to the actual distance between places being measured.

Example: A scale on a map shows 1 inch:20 miles. You measure the distance on the map between Cocoa Beach and Tampa at 7.5 inches. A proportion can be used to determine the actual distance between Cocoa Beach and Tampa. Let *d* equal the actual distance.

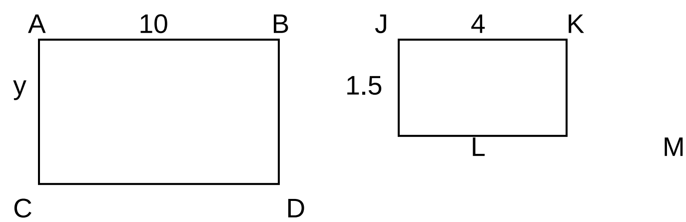
$\frac{1''}{20 \text{ mi.}} = \frac{7.5''}{d}$. Solving for *d*, we get $d = \frac{20 \text{ mi.}(7.5'')}{1''}$ or $d = 150 \text{ mi.}$

Unit 4 Section 2 Worksheet

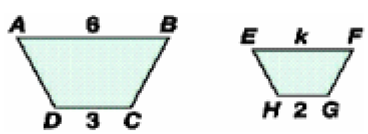


1. $\triangle ABC \sim \triangle TUV$
   a. Calculate the length of segment UV.
   b. Calculate the length of segment TV.

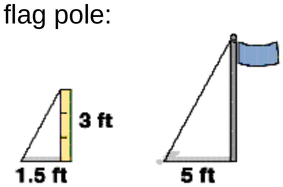
2. Using proportions calculate the missing lengths. Both rectangles are similar.



3. Find the value of k:

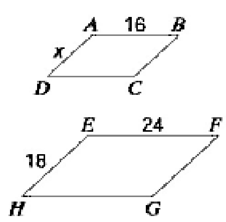


4. Find the height of the flag pole:



5. The scale of a map is 1 in.: 24 mi. About how far is it between two cities that are 3 in. apart on the map?

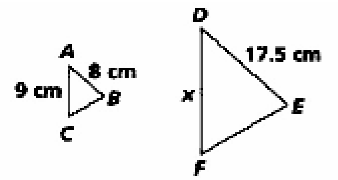
6. Find the value of *x*.



7. A girl who is 4 feet tall casts a shadow that is 6 feet long. The tree next to her casts a shadow that is 12 feet long. How tall is the tree?

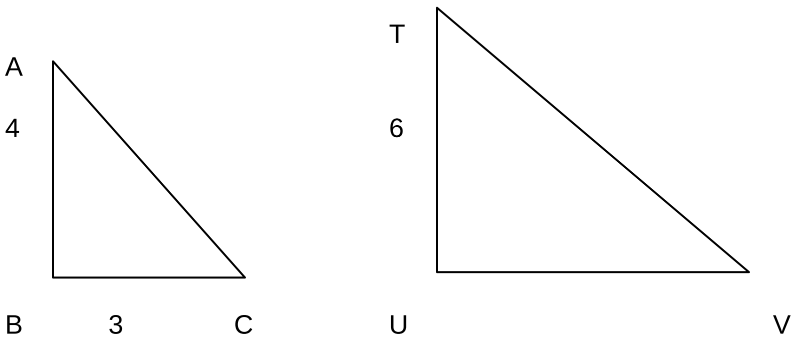
8. The scale on a map is 3 in.: 100 mi. What is the actual distance between two towns that are 7.5 in. apart on the map?

9. Solve for x:



10. A flagpole casts a shadow 102 feet long. A 6 ft. tall man casts a shadow 12 feet long. How tall is the flagpole?

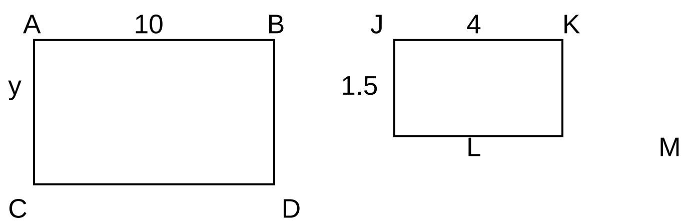
Unit 4 Section 2 Worksheet – ANSWER SHEET



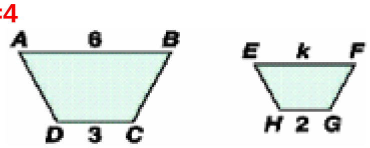
1. $\triangle ABC \sim \triangle TUV$
   a. Calculate the length of segment UV.
   b. Calculate the length of segment TV.

   a. **UV = 4.5**
   b. **TV = 7.5**

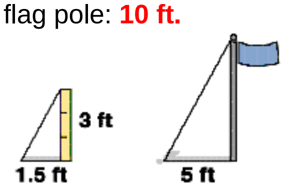
2. Using proportions calculate the missing lengths. Both rectangles are similar. **y= 4.25**



3. Find the value of k: **k=4**

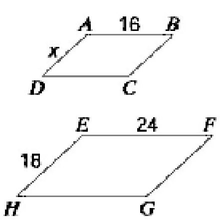


4. Find the height of the flag pole: **10 ft.**



5. The scale of a map is 1 in.: 24 mi. About how far is it between two cities that are 3 in. apart on the map? **72 mi.**

6. Find the value of *x*: **x=12**



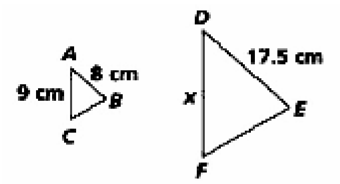
Unit 4 Section 2 Worksheet (continued) – **ANSWER SHEET**

7. A girl who is 4 feet tall casts a shadow that is 6 feet long. The tree next to her casts a shadow that is 12 feet long. How tall is the tree? **The tree is 8 ft. tall**

8. The scale on a map is 3 in.: 100 mi. What is the actual distance between two towns that are 7.5 in. apart on the map?

   250 miles

9. Solve for x: **x= 19.7**



10. A flagpole casts a shadow 102 feet long. A 6 ft. tall man casts a shadow 12 feet long. How tall is the flagpole?

    51 feet

Unit 4 – Solving and Applying Proportions
Section 6 – Probability of Compound Events

Vocabulary:

Independent Events – are events that do not influence each other. The occurrence of one event does not affect the probability of a second event.

Dependent Events – are events that influence each other. The occurence of one event affects the probability of a second event.

OBJECTIVE #1: Finding the probability of independent events.

RULE: Probability of Independent Events

If A and B are independent events, the P(A and B) = P(A) $\cdot$ P(B)

EXAMPLE #1: When two events are independent events, the occurrence of the first event does not affect the probability of the second event.

If you were to roll two 6-sided number cubes, the number that came up on the first cube would not affect the number that comes up on the second cube because they are "independent events". The probability of rolling a 1 on the first cube is 1 out of 6. The probability of rolling a 6 on the second number cube is 1 out of six. The probability of rolling a 1 on the first cube and a 6 on the second cube is found by multiplying each of the probabilities.

Cube 1 – P(1) = $\frac{1}{6}$ Cube 2 – P(6) = $\frac{1}{6}$

The probability of rolling both = P(1) times P(6) = $\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

EXAMPLE #2: If you had a 6-sided number cube and a spinner with four equally-spaced colors on it, the probability of rolling a 3 on the number cube would be $\frac{1}{6}$. The probability of the spinner stopping on one of four specific colors would be $\frac{1}{4}$.

We find the probability of rolling a 3 AND the spinner landing on a specific color by multiplying the two probabilities.

P(3 and red) = P(3) $\cdot$ P(red) = $\frac{1}{6} \cdot \frac{1}{4} = \frac{1}{24}$

NOTE: Some events may seem dependent but, in reality, they are not. For example, if you flipped a coin 4 times and it came up heads all four times, you would be inclined to think that the next time it has to come up tails. The reality is that on the fifth flip, the probability of it coming up heads is still 1 out of 2, or ½. However the probability of flipping a fair coin and it landing on heads five times in a row is found by multiplying the probabilities:

$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{32}$ meaning that the theoretical probability of flipping a coin and it landing on heads five times in a row is 1 out of 32 attempts.

Unit 4 – Solving and Applying Proportions
Section 6 – Probability of Compound Events

<u>OBJECTIVE #2:</u> Finding the probability of dependent events

Dependent events are events in which the outcome of the first event affects the probability of a second event.

RULE: <u>Probability of Two Dependent Events</u>

If A and B are dependent events, the P(A then B) = P(A) $\cdot$ P(B after A)

EXAMPLE #3: Selecting without Replacement.

In a standard deck of 52 cards, there are four Aces. What is the probability of drawing an Ace from the deck on your first card, and then drawing another Ace from the deck on your second card?

The probability of drawing the first Ace = $\frac{4}{52} = \frac{1}{13}$. Once that card is drawn, there are now only 3 aces left in the deck which now has only 51 cards remaining. Now the probability of drawing an Ace is 3 out of 51 or $\frac{3}{51}$.

The probability of drawing two aces on two draws = $\frac{1}{13} \cdot \frac{3}{51} = \frac{3}{661}$

EXAMPLE #4: You are given a bag containing letter tiles, like those in Scrabble, that spell out the word P-R-O-B-A-B-I-L-I-T-Y. What is the probability of a B and then a Y from the bag?

P(B then y) = P(B) $\cdot$P(B then Y)=$\frac{2}{11} \cdot \frac{1}{10} = \frac{2}{110} = \frac{1}{55}$ (Simplified)

Unit 4 – Solving and Applying Proportions
Section 6 – Probability of Compound Events

Unit 4 SECTION 6 WORKSHEET

You roll a red 6-sided number cube and white 6-sided number cube. Find each probability.

1. P(red 2 and white 2)
2. P(red 2 or 3 and white 1)
3. P(red 2 and white 2 or 3)
4. P(red 1 or 2 and white 1 or 2)
5. P(red odd and white odd)
6. P(red and white less than 5)
7. P(red and white less than 6)
8. P(red and white less than 7)
9. P(red 7 and white 6)
10. P(red 1 and white 1)

You select a card at random from those below. Without replacing the card, you choose a second card. Find each probability.

| M | A | T | H | E | M | A | T | I | C | S |

11. P(M then H)
12. P(M then T)
13. P(M then a vowel)
14. P(two vowels)

A drawer contains 4 red sox, 6 blue sox, and 2 black sox. Without looking, you pick a sock. Without replacing it and without looking, you pick a second sock. Find each probability.

15. P(blue, then blue)
16. P(red, then blue)
17. P(red, then black)
18. P(black, then black)

19. You flip a fair two-sided coin four times. What is the probability of flipping a heads each of the four times?

20. You take a five-question multiple choice quiz. You guess on all the questions, selecting one of four answers at random. What is the probability you will answer all five questions correctly?

21. Quality Control Inspectors check a production run of computer chips. Of 800 tested, they find 12 defective chips. Applying that probability to a production run of 48,000 units, how many defective ships will be made?

Unit 4 – Solving and Applying Proportions

Section 6 – Probability of Compound Events

CHAPTER 4 SECTION 6 WORKSHEET - ANSWER SHEET

You roll a red 6-sided number cube and white 6-sided number cube. Find each probability.

1. P(red 2 and white 2) 1/36
2. P(red 2 or 3 and white 1) 1/18
3. P(red 2 and white 2 or 3) 1/18
4. P(red 1 or 2 and white 1 or 2) 1/9
5. P(red odd and white odd) 1/4
6. P(red and white less than 5) 4/9
7. P(red and white less than 6) 25/36
8. P(red and white less than 7) 1
9. P(red 7 and white 6) 0
10. P(red 1 and white 1)1/36

You select a card at random from those below. Without replacing the card, you choose a second card. Find each probability.

M A T H E M A T I C S

11. P(M then H) 1/55
12. P(M then T) 2/55
13. P(M then a vowel) 4/55
14. P(two vowels) 6/55

A drawer contains 4 red sox, 6 blue sox, and 2 black sox. Without looking, you pick a sock. Without replacing it and without looking, you pick a second sock. Find each probability.

15. P(blue, then blue) 5/22
16. P(red, then blue) 6/43
17. P(red, then black) 2/33
18. P(black, then black) 1/66
19. You flip a fair two-sided coin four times. What is the probability of flipping a heads each of the four times? 1/16
20. You take a five-question multiple choice quiz. You guess on all the questions, selecting one of four answers at random. What is the probability you will answer all five questions correctly? 1/1024
21. Quality Control Inspectors check a production run of computer chips. Of 800 tested, they find 12 defective chips. Applying that probability to a production run of 48,000 units, how many defective ships will be made? 720 Defective Chips

$$\frac{\$.0625}{1\text{ oz.}} \bullet \frac{128\text{ oz.}}{1\text{ gal.}} = \frac{\$8.00}{1\text{ gal.}}$$ Seems pretty expensive when compared to a gallon of gas.

b) A cheetah, the world's fastest land animal, ran 300 feet in 2.92 seconds. What was the cheetah's speed in miles per hour?

$$\frac{300\text{ ft.}}{2.92\text{ sec.}} \bullet \frac{1\text{ mi.}}{5280\text{ ft.}} \bullet \frac{60\text{sec.}}{1\text{ min.}} \approx \frac{70\text{ mi.}}{\text{hr.}}$$ Can you see where the common units cancel each other out leaving us with miles per hour?

<u>Solving Proportions:</u>

A proportion is an equation that states two ratios are equal. $\frac{a}{b} = \frac{c}{d}$

The most commonly used method for solving proportions is by using <u>cross products</u> and then solving the equation.

If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$ Example: If $\frac{3}{4} = \frac{9}{12}$, then $3 \bullet 12 = 4 \bullet 9$

<u>Using Cross Products:</u>

Use cross products to solve the proportion $\frac{x}{4} = -\frac{5}{6}$.

$\frac{x}{4} = -\frac{5}{6}$ ← Proportion

$x(6) = (4)(-5)$ ← Write cross products

$6x = -20$ Simplify

$\frac{6x}{6} = \frac{-20}{6}$ Divide both sides by 6

$x = -3.\overline{6}$ Simplify

Solve the problems on the Unit 4 Section 1 Worksheet.

Unit 4 Section 1 Worksheet

Find each unit rate:

1. $48.00 for 5 hours
2. $24 for 9 gallons
3. 850 calories for 4 hours

4. A 39 ounce can of coffee costs $4.99. What is the cost per ounce? (round to the nearest cent)

5. A car travels 210 miles in 3.5 hours. What is it's average speed in miles per hour?

6. Match the correct conversion factor for each situation:

1. $\frac{\text{ft.}}{\text{sec}}$ convert to minutes
2. $\frac{\$2.25}{\text{3 quarts}}$ convert to gallons
3. $\frac{\$3.20}{\text{20 oz}}$ convert to pounds

a) $\frac{\text{4 qts.}}{\text{1 gal.}}$ b) $\frac{\text{1 gal.}}{\text{4 qts.}}$

c) $\frac{\text{1 min.}}{\text{60 secs.}}$ d) $\frac{\text{60 secs}}{\text{1 min.}}$

e) $\frac{\text{1 lb.}}{\text{16 oz.}}$ e) $\frac{\text{16 oz.}}{\text{1 lb.}}$

Solve each proportion:

7. $\frac{8}{12}=\frac{x}{3}$ 8. $\frac{n}{25}=\frac{72}{100}$ 9. $\frac{8}{5}=\frac{n}{25}$ 10. $\frac{m+2}{4m}=\frac{9}{6}$

11. $\frac{2}{14}=\frac{9}{y}$ 12. $-\frac{3}{8}=\frac{t}{12}$ 13. $\frac{5}{4}=-\frac{25}{b}$ 14. $\frac{80}{x}=\frac{5}{3}$

Complete each statement:

15. $4/lb. = ___ cents/oz. 16. 6 cm./hr. = ___ m/week

17. 1 mi. in 5 min. = ___ mi./hr. 18. 1 qt./hr. =___ gal./week

19. You are riding your bicycle and it takes you 20 minutes to go 4 miles.
    a) How long will it take you to go 14 miles?
    b) What is your speed in miles/hour?

Unit 4 Section 1 Worksheet - ANSWERS

Find each unit rate:

1. $48.00 for 5 hours
$9.60/hr

2. $24 for 9 gallons
$$2.67/gal

3. 850 calories for 4 hours
212.5 cal/hr

4. A 39 ounce can of coffee costs $4.99. What is the cost per ounce? (round to the nearest cent)
$0.13/oz

5. A car travels 210 miles in 3.5 hours. What is it's average speed in miles per hour?
60 mi/hr

6. Match the correct conversion factor for each situation:

1. $\frac{\text{ft.}}{\text{sec}}$ convert to minutes

2. $\frac{\$2.25}{\text{3 quarts}}$ convert to gallons

3. $\frac{\$3.20}{\text{20 oz}}$ convert to pounds

a) $\frac{\text{4 qts.}}{\text{1 gal.}}$ b) $\frac{\text{1 gal.}}{\text{4 qts.}}$

c) $\frac{\text{1 min.}}{\text{60 secs.}}$ d) $\frac{\text{60 secs}}{\text{1 min.}}$

e) $\frac{\text{1 lb.}}{\text{16 oz.}}$ e) $\frac{\text{16 oz.}}{\text{1 lb.}}$

#1-d, #2-a, #3-e

Solve each proportion:

7. $\frac{8}{12}=\frac{x}{3}$ 2

8. $\frac{n}{25}=\frac{72}{100}$ 18

9. $\frac{8}{5}=\frac{n}{25}$ 40

10. $\frac{m+2}{4m}=\frac{9}{6}$ 0.4

11. $\frac{2}{14}=\frac{9}{y}$ 63

12. $-\frac{3}{8}=\frac{t}{12}$ 4.5

13. $\frac{5}{4}=-\frac{25}{b}$ 20

14. $\frac{80}{x}=\frac{5}{3}$ 48

Complete each statement:

15. $4/lb. = _25_ cents/oz.

16. 6 cm./hr. = 10.08 m/week

17. 1 mi. in 5 min. = 12 mi./hr.

18. 1 qt./hr. = 42 gal./week

19. You are riding your bicycle and it takes you 20 minutes to go 4 miles.
a) How long will it take you to go 14 miles? 70 minutes
b) What is your speed in miles/hour? 12 miles/hr