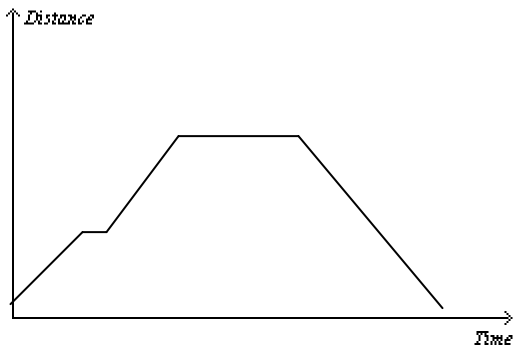
Unit 5 – Graphs and Functions

Section 1 – Relating Graphs to Events

OBJECTIVE #1 – Interpreting, Sketching, and Analyzing Graphs
Just as you can use an equation, an inequality, or a proportion to make a statement about a variable, you can use a graph to show the relationship between two variables. For example, you can use a graph to show changes in a quantity over time.

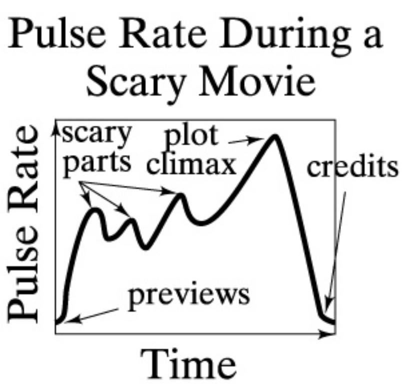
Example #1: Interpreting Graphs

A student is leaving the house and driving his car to school. The graph below relates the distance traveled from home to the amount of time the students travels. In the first section of the graph, the student leaves the house and begins driving. The second, flat portion of the graph could be stopping at a friend's house or at a stoplight. The student then drives on until reaching school and parks the car. The final downward portion of the graph is when the student leaves class and drives the car home. Notice there are no stops along the way.



Example #2: Sketching a Graph

A movie producer wants to get a measure of how scary his movie is so he measures and sketches the pulse rates of people at a special screening of the movie. Label each section.



Unit 5 – Graphs and Functions
Section 4 – Writing a Function Rule

OBJECTIVE #1: WRITING FUNCTION RULES

You should already be familiar with arithmetic sequences where each successive number in a sequence is the result of adding or subtracting a fixed amount and you use that to determine the next numbers in the series. You should also be familiar with geometric series where each successive number in a series is a result of multiplying the previous number by a constant ratio.

Arithmetic Sequence: 1,3,5,7,….. What are the next three numbers in the sequence? What is the rule to determine the next numbers in the sequence? The next three numbers in the sequence are 9,11,13. The rule for this sequence would be "start at 1 and add 2".

We can analyze information and data in tables to determine a pattern and write a function rule that describes the relationship between the independent (input) and dependent (output) variables. If we've analyzed the information properly and written an accurate function rule, the rule can often be used to predict outcomes.

<u>EXAMPLE #1:</u> Writing a Rule from a Table
Write a rule for each table.

a) As we look at the data in the table we must ask ourselves what is happening to the x-value to get to the f(x) value. What is done to 1 to get to 4? to 2 to get to 5? …?

x	f(x)
1	4
2	5
3	6
4	7

There are two possibilities to get from 1 to 4. One is to add 3. The other is to multiply by 4. When we look at the next input value we can rule out multiplying by 4, since $2 \cdot 4 \neq 5$, therefore we can define the the function rule as $f(x)=x+3$. Using this function rule, we can determine the f(x) or output value for any input value of x.

b)

x	y
1	1
2	4
3	9
4	16

As we analyze the table and ask ourselves what we can do to get 1 to 1, to get 2 to 4, to get 3 to 9, …. We see the pattern where x is multiplied by itself to get y, the output value. From this, we can write a function rule for this data. If y equals x times itself, then the function rule would be $y=x^2$. Using this function rule, we can determine the output value for any value of x.

EXAMPLE #2: REAL-WORLD EXAMPLE

A contractor specializes in putting in driveways, patios, and pool aprons. A big part of his cost is concrete. He knows it costs $150.00 for the truck to make a local trip plus he is charged $23.00 per cubic yard of concrete. Write a function the contractor can use to estimate the concrete costs for any job.

Relate: Total concrete cost is $150 + $23 times number of yards of concrete

Define: Let n = number of yards of concrete
Let C(n) = total concrete costs

Write: C(n) = $150 + $23 • n

The function rule $C(n)=150+23n$ describes the concrete costs as a function of yards of concrete needed.

EXAMPLE #3: REAL-WORLD EXAMPLE

After a hurricane, you decide to make some extra money by cutting up fallen trees and hauling them away. You rent a chain saw for $90 per day and a stump grinder for $145 per day. You charge $200 for each tree you remove. Write a function rule to describe your profit as a function of the number of trees removed.

Relate: Total Profit = $200 times trees removed minus rental cost of equipment.

Define: Let t = trees removed
Let P(t) = total profit

Write: P(t) = 200 • t - (90+145)

The function rule $P(t)=200t-235$ describes your profit for the day as a function of the number of trees removed.

CHAPTER 5 SECTION 4 WORKSHEET

Write a function rule for each table:

1.

x	y
1	2
2	4
3	6
4	8

2.

x	y
1	0.5
2	1
3	1.5
4	2

3.

x	f(x)
1	-2
2	-4
3	-6
4	-8

4.

x	f(x)
-2	4
-1	1
0	0
1	1
2	4

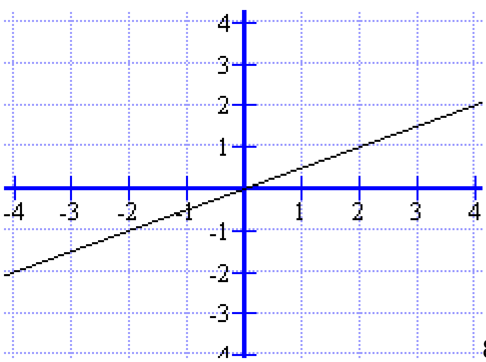
5.

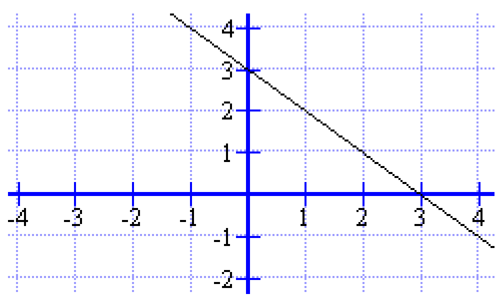
L	ml
.5	500
1.0	1000
1.5	1500
2.0	2000

6.

Inches	Centimeters
1	2.54
2	5.08
3	7.62
4	10.16

Make a table and use the table to write a function rule.

7. 

8. 

9. A car rental company charges $29.00 per day plus $.18 per mile. Write a function rule for calculating the cost of renting the car for a day and driving m miles.

a) What is the cost of driving 90 miles? What is the cost of driving 142 miles?

b) You return the car to the company and your bill, excluding tax, comes to $39.26. How far did you drive?

Unit 5 SECTION 4 WORKSHEET- ANSWER SHEET

Write a function rule for each table:

1. y=2x

x	y
1	2
2	4
3	6
4	8

2. y=0.5x

x	y
1	0.5
2	1
3	1.5
4	2

3. f(x)=-2x

x	f(x)
1	-2
2	-4
3	-6
4	-8

4. $f(x)=x^2$

x	f(x)
-2	4
-1	1
0	0
1	1
2	4

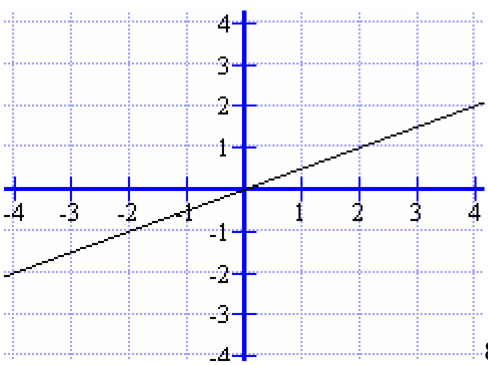
5. f(ml)=1000x

L	ml
.5	500
1.0	1000
1.5	1500
2.0	2000

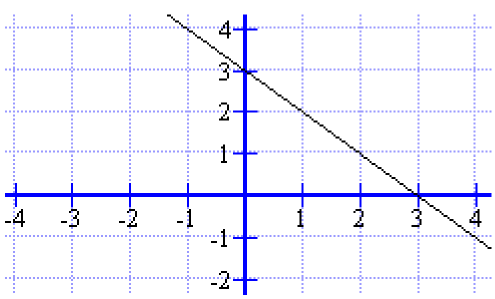
6. f(cm)= 2.54x

Inches	Centimeters
1	2.54
2	5.08
3	7.62
4	10.16

Make a table and use the table to write a function rule.

7. 

y=0.5x

8. 

y=-x+3

9. A car rental company charges $29.00 per day plus $.18 per mile. Write a function rule for calculating the cost of renting the car for a day and driving m miles.

a) What is the cost of driving 90 miles? What is the cost of driving 142 miles?

b) You return the car to the company and your bill, excluding tax, comes to $39.26. How far did you drive?

C(m) = 29 + .18(m)

a) $45.20 $54.56

b) 57 miles

Unit 5 – Graphs and Functions

Section 3 – Function Rules, Tables, and Graphs

Vocabulary:

<u>Independent Variable</u>: Independent variables are the inputs or domain values used to evaluate a function
or develop a table.

<u>Dependent Variable</u>: Dependent variables are the outputs (range values) or the corresponding values for
the particular independent variable input.

<u>OBJECTIVE #1: MODELING FUNCTIONS</u>

Functions can be modeled in three ways. You can use function rules, which show how the variables are related.
You can make a table which identifies specific input and output values for a function, or, you can make a graph
to give a visual picture of the function.

When making a graph, the input values (independent variables) are graphed on the horizontal axis. The output values (dependent variables) are graphed on the vertical axis. The input and output values are used as ordered pairs to plot points. Once the points are plotted, join the points with a line or smooth curve to give a general picture of the function.

<u>EXAMPLE #1:</u> Three Views of a Function

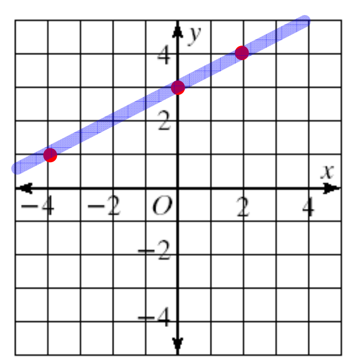
Model the function rule $y = \frac{1}{2}x + 3$ using a table of values and a graph.

Step 1 – Make a table and choose the input values for x. Evaluate for y.

Step 2 – Plot points for the ordered pairs.

Step 3 – Join the points to form a line.

x	$\frac{1}{2}(x)+3$	y
-4	$\frac{1}{2}(-4)+3$	1
0	$\frac{1}{2}(0)+3$	3
2	$\frac{1}{2}(2)+3$	4



As you can see from the graph, this function rule produces a straight line. This particular function rule is in the form that is called a linear equation because it generates a straight line.

Check Understanding: Make a table and graph for the function rule f(x)=2x+4 .

When you draw a graph for a real-world situation, you have to make sure you choose the appropriate intervals for the units on the axes. Be sure the intervals are equal. Also, if the data are positive numbers, use only the first quadrant.

EXAMPLE #2: Real-World Problem Solving

New Year's Eve Party – Suppose your group wants to sponsor a New Year's Eve Party. You check with a restaurant that will supply their facility, dinner and provide live music and party favors for a cost of $1000 plus $50.00 per couple. The total cost depends on the number of couples attending. Use the function rule P(c) = 1000 + 50c to make a table of values and a graph.

c	P(c)=1000+50c	(c,P(c))
10	1000+50(10)	(10,1500)
20	1000+50(20)	(20,2000)
30	1000+50(30)	(30,2500)
40	1000+50(40)	(40,3000)

Check Understanding: a) Another restaurant charges $1500 for the evening but only $25.00 per couple. Write a function rule for this second restaurant and make a table and graph. How many couples need to attend for this restaurant to become cheaper than the first one?

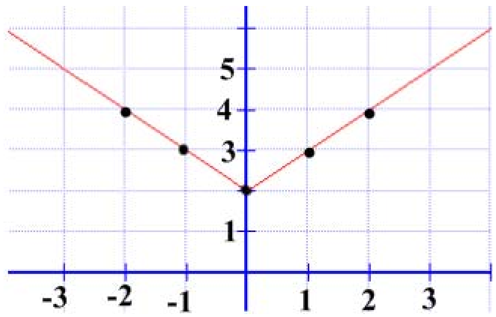
Examples #1 and #2 have been Linear Functions. The function rule has been in linear equation form.

Some functions have graphs that are not straight lines, but you can still graph the function as long as you know its rule. After you have graphed the ordered pairs that you have calculated from the rule, connect the points with a smooth line or curve. We will look at several more function families.

EXAMPLE 3: Absolute Value Functions

Graph the function y=lxl + 2.

x	y=lxl+2	(x,y)
-2	l-2l+2	(-2,4)
-1	l-1l+2	(-1,3)
0	l0l+2	(0,2)
1	l1l+2	(1,3)
2	l2l+2	(2,4)



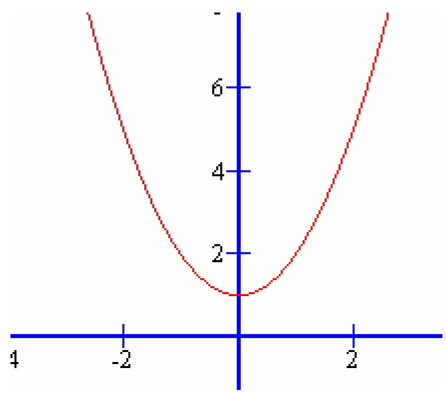
Check Understanding: Make a table of values and graph the function y=lxl-1

EXAMPLE #4: Parabolic Curve

Functions in which the input variable (independent variable) has an even-numbered exponent produce a shape called a parabola.

Graph the function $f(x)=x^2+1$

x	$f(x)=x^2+1$	(x,y)
-2	4+1=5	(-2,5)
-1	1+1=2	(-1,2)
0	0+1=1	(0,1)
1	1+1=2	(1,2)
2	4+1=5	(2,5)



Check Understanding: Make a table and graph the function $f(x)=x^2-1$.

Unit 5 SECTION 3 WORKSHEET

Model each rule with a table and graph:

1. $f(x)=-2x+1$

2. $f(x)=-2x-3$

3. $f(x)=\frac{1}{3}x$

4. $y=3x$

5. $f(x)=|x|$

6. $f(x)=|x|+2$

7. $f(x)=|x|-2$

8. $f(x)=x^2$

9. $f(x)=x^2+1$

10. $y=x^2-3$

11. $y=\left|\frac{1}{2}x\right|+1$

12. $y=-|x+2|$

Graph each function:

13. $f(x)=1-x^2$

14. $f(x)=-4x^2$

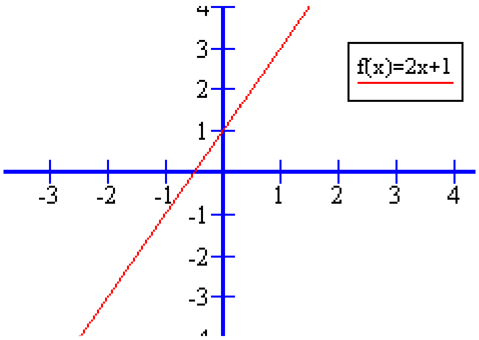
15. $f(x)=-|x-2|$

16. It costs $150 to join a health club, plus a monthly fee of $75.00. Write a function rule for the cost of the membership and make a graph. What is the cost of membership for 6 months? How many months can you be a member if you have $700.00.

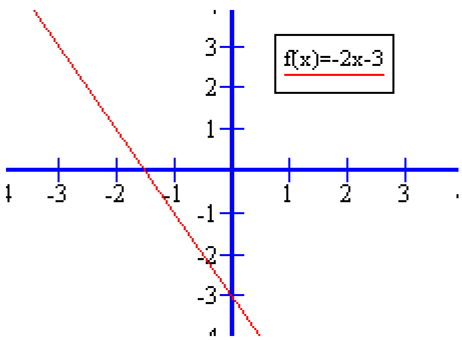
Unit 5 SECTION 3 WORKSHEET – ANSWER SHEET

Model each rule with a table and graph: Student answers in table may vary.

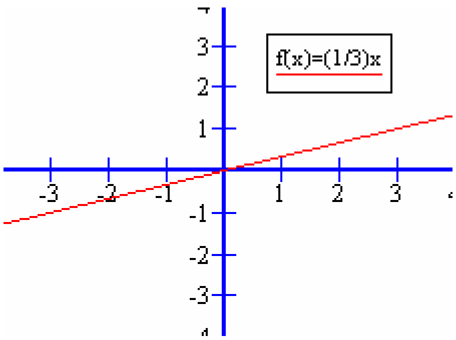
1. f(x)=-2x+1



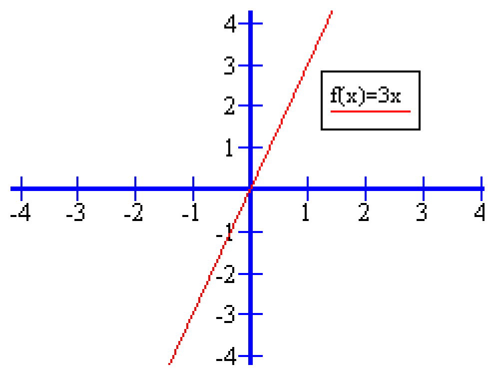
2. f(x)=-2x-3



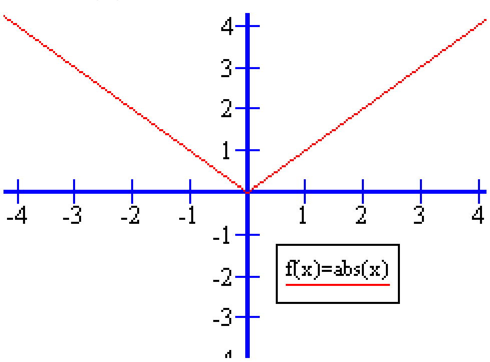
3. $f(x)=\frac{1}{3}x$



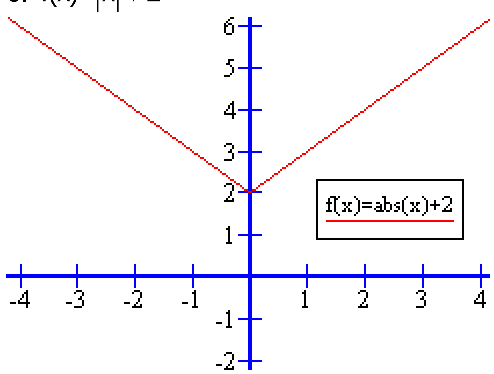
4. y=3*x*



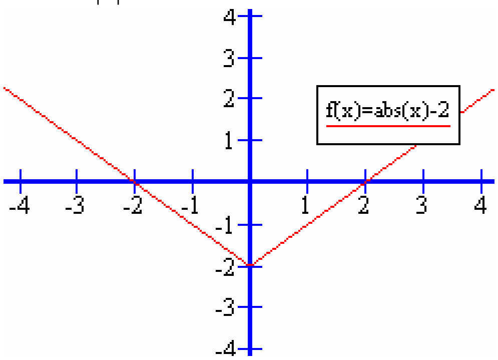
5. $f(x)=|x|$



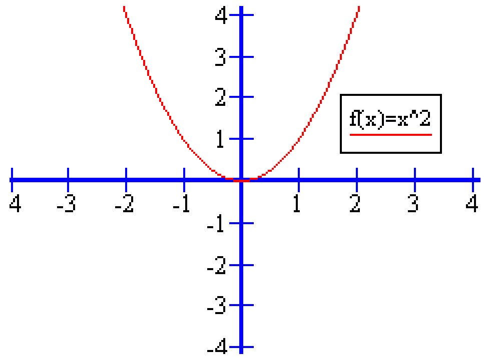
6. $f(x)=|x|+2$



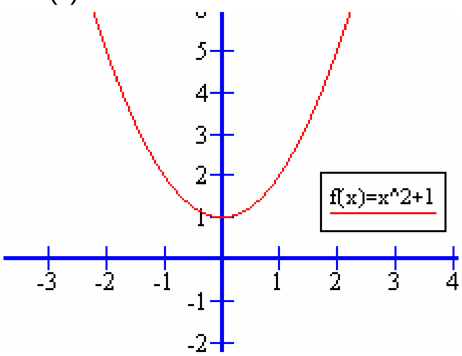
7. $f(x)=|x|-2$



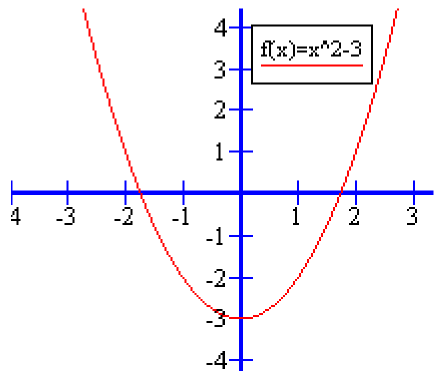
8. $f(x)=x^2$



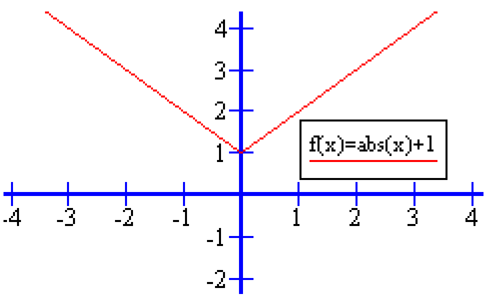
9. $f(x)=x^2+1$



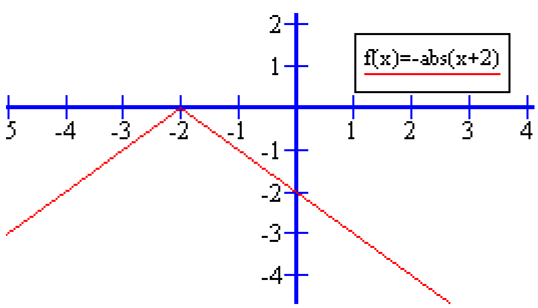
10. $y=x^2-3$



11. $y=\left|\frac{1}{2}x\right|+1$

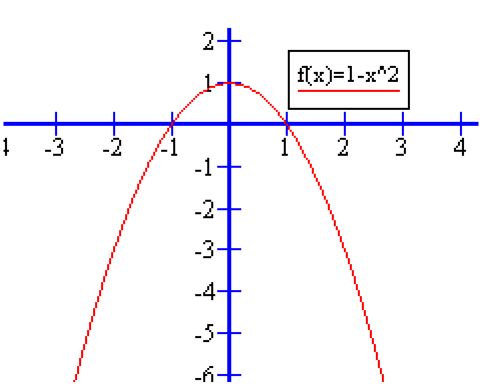


12. $y=-|x+2|$

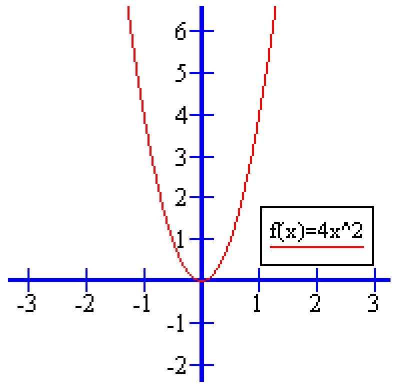


Graph each function:

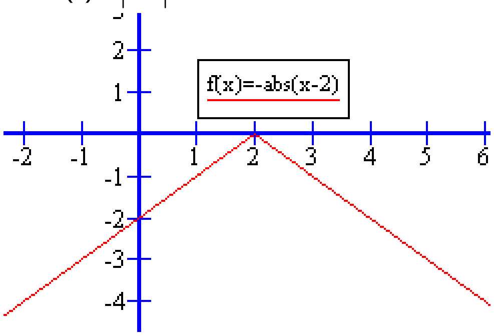
13. $f(x)=1-x^2$



14. $f(x)=-4x^2$

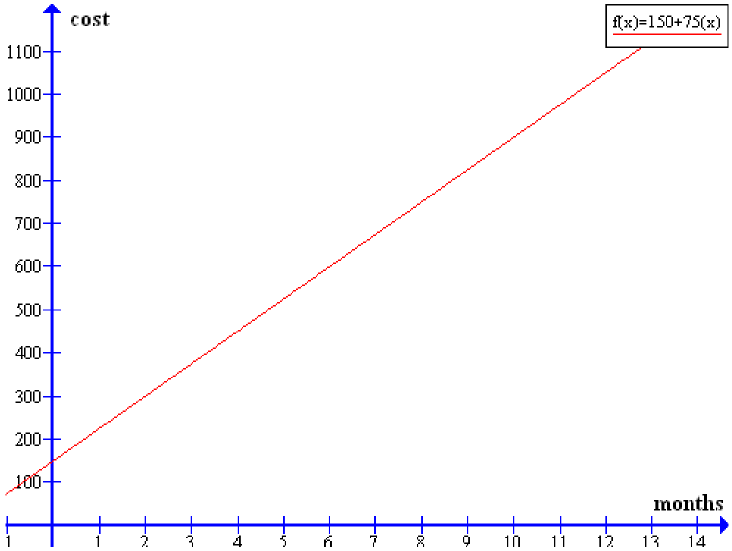


15. $f(x)=-|x-2|$



16. It costs $150 to join a health club, plus a monthly fee of $75.00. Write a function rule for the cost of the membership and make a graph. What is the cost of membership for 4 months? How many months can you be a member if you have $700.00.

a. f(m) = 150 + 75(m)
c. $450.00
d. 7 months


cost
f(x)=150+75(x)
months
1100
1000
900
800
700
600
500
400
300
200
100
1 2 3 4 5 6 7 8 9 10 11 12 13 14

Unit 5 – Graphs and Functions
Section 2 – Relations and Functions

Vocabulary:

Relation – a relation is a set of ordered pairs.

Domain – a domain is the set of first coordinates of the ordered pairs.

Range – the range is the set of second coordinates of the ordered pairs.

Function – a function is a relation that assigns exactly one (and only one) value in the range to each value in the domain.

Vertical-line Test – if a vertical line passes through more than one point of the graph, the relation is not a function.

Function rule – a function rule is an equation that describes a function.

Function notation – a function is in function notation when you use *f(x)* to indicate the outputs. You read as "*f* of *x*" or "*f* is a function of *x*". The notations *g(x)* and *h(x)* also indicate functions of *x*.

OBJECTIVE #1: IDENTIFYING RELATIONS AND FUNCTIONS

A relation is a set of ordered pairs. When you graph a point (x,y) on a coordinate plane you are graphing an ordered pair. To have a relation you have to have a "set" of ordered pairs. In other words, you need more than one. The ordered pairs in the table below form a relation.

Troop #	# of Scouts	# of Cookie Boxes Sold
85	18	237
96	21	235
23	23	143
62	21	429
58	20	285

The domain of the relation is the set of first coordinates of the ordered pairs. In this case, the domain would be the number of scouts in each troop. The range is the set of second coordinates (the number of boxes sold).

We can write the ordered pairs to indicate they are a relation by enclosing them in brackets.

{(18,237), (21,235), (23,143), (21,429), (20,285)}

EXAMPLE #1: Find the domain and range of the ordered pairs listed above for the sale of the cookies.

domain: {18,20,21,23} List the values in order. Do not repeat values.
range: {143, 235, 237, 285, 429}

Check Understanding: Find the domain and range of the relation represented by the data in the table.

{-3, -1, 3}, {-1,1,2,3}

Domain	Range
-3	2
-1	3
3	-1
-1	1
3	3

A function is a relation that assigns exactly one (and only one) value in the range to each value in the domain.

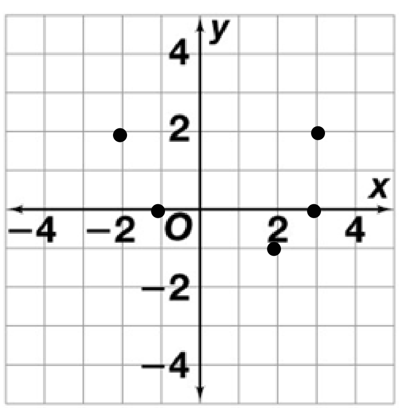
Tip: One way you can tell whether a relation is a function is to analyze the graph of the relation using the **vertical-line test**. If any vertical line passes through more than one point on the graph, the relation is NOT a function.

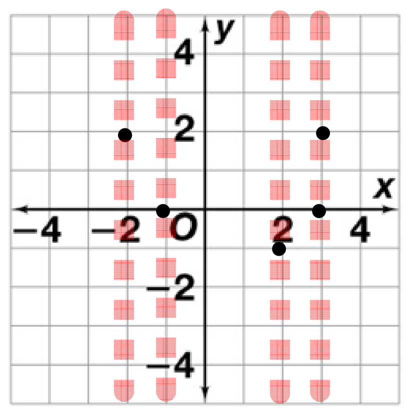
EXAMPLE #2: Using the Vertical-Line Test

Determine whether the relation {(2,-1), (3,0), (-1,0), (-2,2), (3,2) is a function.

Step 1 – Graph the ordered pairs on a coordinate plane.

Step 2 – Pass a pencil across the graph as shown by the dotted line..





A vertical line would pass through both (3,0) and (3,2) so this relation is not a function.

Check Understanding: Use a vertical-line test to determine whether or not each relation is a function.
a) {(1,2), (2,3), (0,1), -1,0)} b) {(-2, -1), (-1,0), (4,2), (-1, 3)}

Is a function Is not a function.

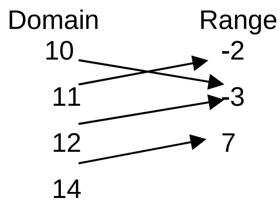
Using a vertical-line test is an easy way to look at a graph of ordered pairs and tell whether or not the relation is a function.

Another way to tell whether a relation is a function is by making a **mapping diagram**. List the domain values in a column (in order with no repeating of numbers) and list the range values in a column. Again, list them in order and list each number only once. We will draw arrows from each domain value to its' range value. If the relation is a function, there will be only one range value for each domain value.

EXAMPLE #3: Using a Mapping Diagram

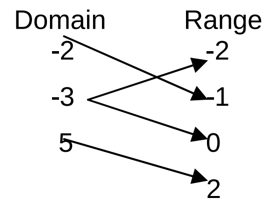
Determine whether each relation is a function.

A) {(10,-3), (11,-2), (12,-3), (14,7)}



This relation is a function. There is no value in the domain that corresponds to more than one value in the range.

B) {(-3,-2), (-2,-1), (5,2), (-3,0)}



This is NOT a function. The domain value -3 corresponds to two range values.

Note: There can be more than one domain value that corresponds to a range value; BUT, each particular domain value can only correspond to one particular range value is the relation is a function.

Check Understanding: Use a mapping diagram to determine whether each relation is a function.

a) {(4,-2), (3,-1), (6, 2), (3,3), (5,1)}
Not a function

b) {(-1,-1), (-2,-3), (1,3), (0,1), (2,5)}
Function

OBJECTIVE #2: EVALUATING FUNCTIONS

A **function rule** is an equation that describes a function. There are many different types of functions; but, in all cases, you can think of a function rule as an input-output machine. The domain is the set of input values you will plug into the input-output machine. The function rule, the input-output machine, outputs the range values. We have to keep in mind that for each domain (input) value there is exactly one range (output) value.

If you know the input values, you can use a function rule to determine the output values.

These words and notations are often used with functions.

Domain	**Range**
Input	Output
x	f(x)
x	y

y = 2x + 5 → This is a function rule.

^ output, ^ input — We will input values for x and yield output values for y.

Input	Output
x	y
1	7
2	9
3	11

Another way to write the function y = 2x + 5 is f(x) = 2x + 5. A function rule is written in function notation when it is written using f(x) to indicate the outputs. You read f(x) as "f of x" or "f is a function of x". You may also see them written using other letters, such as g(x), h(x), and k(x).

EXAMPLE #4: Evaluating a Function Rule

a) Evaluate f(t) = -2t – 20 when t = 7

f(t) = -2t – 20

f(7) = -2(7) – 20 Substitute 7 for t

f(7) = -14 – 20 Simplify

f(7) = -34

b) Evaluate $f(x) = -2x^2 + 14$ for x = -3

$f(x) = -2x^2 + 14$

$f(-3) = -2(-3)^2 + 14$ Substitute -3 for x

$f(-4) = 16$ Simplify the power

$f(-3) = -18 + 14$ Simplify

$f(-3) = -4$

Check Understanding: Evaluate each function rule for x = 2.3.

a) $y = 3x + 1$ 7.9

b) $f(x) = -x + 3$ 0.7

c) $h(x) = x^2 - 2$ 3.29

With a function rule and a given domain values, we can find the range of the function. After computing the range values, write the values in order from least to greatest.

EXAMPLE #5: Finding the Range

Evaluate the function rule f(c)= $-3c + 4$ to find the range of the function for the domain {3,1,-4}.

$f(c) = -3c + 4$	$f(c) = -3c + 4$	$f(c) = -3c + 4$
$f(3) = -3(3) + 4$	$f(1) = -3(1) + 4$	$f(-4) = -3(-4) + 4$
$f(3) = -5$	$f(1) = 1$	$f(-4) = 16$

The range is {-5, 1, 16}.

Check Understanding: Find the range of the function for the domain {-3, 1, 2}

a) $f(n) = n - 4$ {-7, -3, -2}

b) $f(x) = -3x$ {9, -3, -6}

c) $f(r) = \pi r^2$ {28.26, 3.14, 12.25}

Unit 5 SECTION 2 WORKSHEET

Find the domain and range of each relation.

1. {(3,4), (4,5), (5,7), 6,8), (7,9)}
2. {(2,-3), (1,6), (3,-7), (-2,5), (4,6)}
3. {(-2,-4), (0,3), (1,-4), (3,3), (4,-2)}
4. {(1.2, 2.4), (1.6, 3.2), (-1,-2), (2.5,5)}

Use the vertical-line test to determine whether each relation is a function.

5. {(-4,-2), (0,2), (3,5), (4,6), (8,10)}
6. {(-1,2), (-2,5), (0,1), (-2, 3), (2,5)
7. {(5,0), (0,5), (5,1), (1,5)
8. {(-2,8), (-3, 8), (2,8), (5,8)

Use a mapping diagram to determine whether each relation is a function.

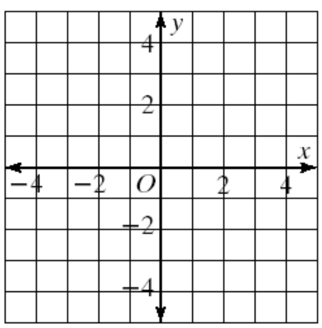
9. {(3,6), (2, 3), (-1,2), (-2, 5), (2,6)}
10. {(-3,1), (-1,4), (2,8), (-3,6, (-2,4)}
11. {(1,1), (2,4), (0,0), (-1,1), (-2,4)}
12. {(1.2, 2.4), (2, 4), (3, 6), (-6, -3), (2,-4)

Evaluate each function rule for x = -4.

13. $y = x + 3$
14. $y = 6x - 7$
15. $f(x) = x^2 + 1$
16. $f(x) = -11 - x$

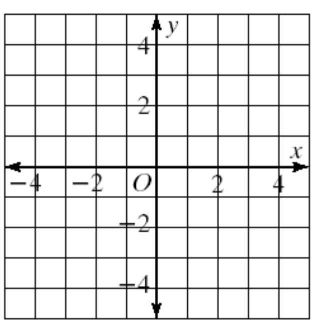
Graph each relation. Is the relation a function? Explain.

16.



x	y
0	2
1	3
2	4
3	5
2	0
1	1

17.



x	Y
-3	3
-2	2
-1	1
0	0
1	1

Unit 5 Section 2 WORKSHEET – ANSWER SHEET

Find the domain and range of each relation.

1. {(3,4), (4,5), (5,7), 6,8), (7,9)} Domain {3, 4, 5, 6, 7} Range {4, 5, 7, 8, 9}
2. {(2,-3), (1,6), (3,-7), (-2,5), (4,6)} Domain {-2,1, 2, 3, 4} Range {-7, -3, 5, 6}
3. {(-2,-4), (0,3), (1,-4), (3,3), (4,-2)} Domain {-2, 9, 1, 3, 4} Range {-4, -2, 3}
4. {(1.2, 2.4), (1.6, 3.2), (-1,-2), (2.5,5)} Domain {-1, 1.2, 1.6, 2.5} Range {-2, 2.4, 3.2, 5}

Use the vertical-line test to determine whether each relation is a function.

5. {(-4,-2), (0,2), (3,5), (4,6), (8,10)} Yes, it is a function.
6. {(-1,2), (-2,5), (0,1), (-2, 3), (2,5) No.
7. {(5,0), (0,5), (5,1), (1,5) No.
8. {(-2,8), (-3, 8), (2,8), (5,8) Yes.

Use a mapping diagram to determine whether each relation is a function.

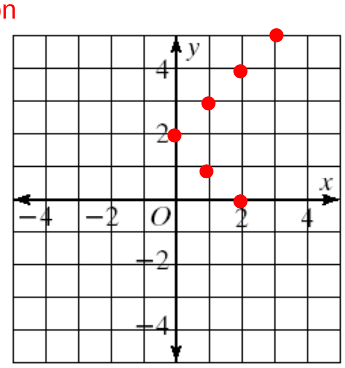
9. {(3,6), (2, 3), (-1,2), (-2, 5), (2,6)} No.
10. {(-3,1), (-1,4), (2,8), (-3,6, (-2,4)} Yes.
11. {(1,1), (2,4), (0,0), (-1,1), (-2,4)} Yes.
12. {(1.2, 2.4), (2, 4), (3, 6), (-6, -3), (2,-4) No.

Evaluate each function rule for x = -4.

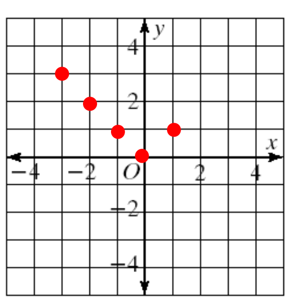
13. $y = x + 3$ y = -1
14. $y = 6x - 7$ y = -31
15. $f(x) = x^2 + 1$ f(x) = 17
16. $f(x) = -11 - x$ f(x) = -7

Graph each relation. Is the relation a function? Explain.

16. No, not a function



17. Yes, a function



Unit 5 – Graphs and Functions
Section 5 – Direct Variation

Vocabulary:

Direct Variation – A function is said to be a direct variation when the output is the product of the input times a coefficient that is constant for all inputs. The function is expressed as y=kx where k is the "constant of variation".

Constant of Variation – expressed by the letter "k" and is a coefficient of the input values for a function. "k" does not change as input values change.

An example of direct variation and a function using a constant is the formula used to calculate the diameter of a circle. $c=\pi d$ The value of pi remains constant.

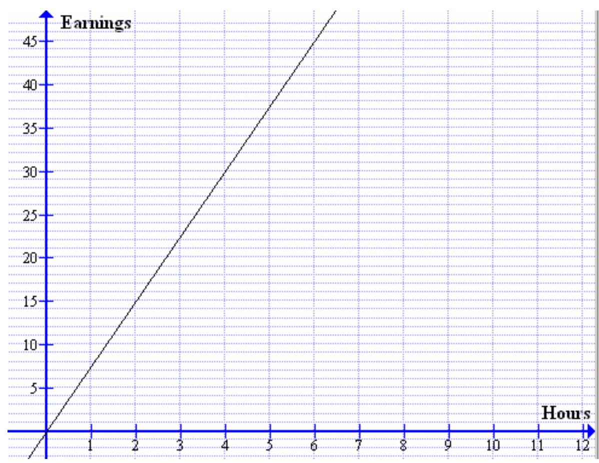
OBJECTIVE #1: WRITING THE EQUATION OF A DIRECT VARIATION

Let us suppose you have a job that paid you an hourly wage of $7.50 per hour. The amount you earn varies directly with the number of hours you work. Here are three ways you can model the relationship between your earnings E(h) and the number of hours you work h.

1. Table

h Hours Worked	E(h) Earnings
1	7.50
2	15.00
3	22.50
4	30.00
5	37.50

2. Graph



3. Function Rule E(h)=7.50h

Investigate: a) As the number of hours worked doubles, what happens to the amount of earnings?
The amount doubles

b) Find the ratio for each pair of data in the table. $\frac{7.50}{1}$

c) For every increase of 1 hour on the horizontal axis, what is the increase on the y-axis?
7.50

Notice that the answers to b and c are the same. The amount of earnings varies directly with the number of hours worked therefore we have a "direct variation" and the constant of variation is 7.50.

For $y=kx$, y is a function of x. If x = 0, then y = 0, so the graph of a direct variation is a line that passes through (0,0). To tell whether an equation represents a direct variation, solve the equation for y. If the equation can be written in the form y = kx, it represents a direct variation.

EXAMPLE #1: Is an equation direct variation? If it is, find the constant of variation.

a) $4x+3y=0$

$3y=-4x$ Subtract 4x from both sides

$y=-\frac{4}{3}x$ Divide both sides by 3

The equation has the form y=kx, so the equation is a direct variation.

The constant of variation is $-\frac{4}{3}$

b) $4x+3y=5$

$3y=5-4x$ Subtract 4x from both sides

$y=\frac{5}{3}-\frac{4}{3}x$ Divide both sides by 3

The equation cannot be written in the form y=kx. It is NOT a direct variation.

To write an equation for a direct variation, you must first find the constant of variation k using a point other than (0,0) that lies on the graph of the equation. Then use the value of k to write the equation.

How do you do this?

EXAMPLE #2:

Write the equation for a direct variation that includes the point (4,5).

1. Start with the function form of a direct variation $y = kx$
2. Substitute the 4 for x and the 5 for y $5 = k(4)$
3. Divide both sides by 4 to solve for k $\frac{5}{4}=k$
4. Write an equation substituting $\frac{5}{4}$ $y=\frac{5}{4}x$

An equation of the direct variation is $y=\frac{5}{4}x$

You can use a direct variation to describe a real-world situation in which the dependent variable varies directly with the independent variable.

EXAMPLE #3:

Mapping the ocean floor involves sending out sonar pings and measuring the time it takes for the sound to be reflected back to the sensor. If it takes 2 seconds for the ping to be returned, the ocean bottom is about 1500 meters deep. Write an equation for the relationship between time and distance.

Relate: The distance varies directly with time. When x = 2 secs, y = 1500 m.

Define: Let x = the number of seconds it takes the sound to be returned.
Let y = the distance in meters from the ocean floor.

Write:		
	$y = kx$	Use the general form of a direct variation
	1500=k(2)	Substitute 1500 for y and 2 for x
	$\frac{1500}{2} = k$	Divide both sides by 2
	750 = k	We have a constant of variation.

The equation $y = 750x$ relates the time in seconds for the return of a sonar ping to the depth of the ocean floor.

OBJECTIVE #2: RATIOS, PROPORTIONS, AND TABLES

In the direct variation function y = kx, k is constant and can be expressed as the value of y divided by the value of x. $k = \frac{y}{x}$. When two sets of data vary directly, the ratio $\frac{y}{x}$ is the constant of variation. It will be the same for every data pair.

EXAMPLE #4:

Given that for a direct variation function, the ratio $\frac{y}{x}$ is constant, we can evaluate the data in table to tell whether y varies directly with x. If it does, write an equation for the direct variation.

a)

x	y	$\frac{y}{x}$
-4	3	$\frac{3}{-4} = -0.75$
1	-0.75	$\frac{-0.75}{1} = -0.75$
4	-3	$\frac{y_1}{x_1} = \frac{y_2}{x_2}$
6	-4.5	$\frac{-4.5}{6} = -0.75$

The constant of variation is -0.75.
The equation is y = -0.75x
If the ratios from any row of the data came out differently, we would not have a direct variation.

b)

x	y	$\frac{y}{x}$
2	-1	$\frac{-1}{2} = -0.5$
4	2	$\frac{2}{4} = 0.5$
6	3	$\frac{3}{6} = 0.5$
8	2	$\frac{2}{8} = 0.25$

No, the ratio $\frac{y}{x}$ is not the same for all pairs of data; therefore, we do not have direct variation.

In a direct variation, the ratio $\frac{y}{x}$ is the same for all pairs of data where x $\neq 0$. This means the proportion $\frac{y_1}{x_1} = \frac{y_2}{x_2}$ is true for the ordered pairs (x_1, y_1) and (x_2, y_2) where neither x value is zero.

Unit 5 SECTION 5 WORKSHEET

For the data in each table, tell whether y varies directly with x. If it does, write an equation for the direct variation.

1.

x	y
3	4.8
5	8
8	12.8

2.

x	y
-2	.5
2	-.5
4	-2

3.

x	y
6	4.5
9	6
12	9

4.

x	y
1	3
2	2
3	1

Write an equation for the direct variation that includes the given point.

5. (2,4)

6. (0.5, 3)

7. (-2, $\frac{1}{8}$)

8. $(-3,5)$

9. (1.2,7.2)

10. (4.8,3.6)

11. (2.4,4.2)

12. (3.4,5.1)

The ordered pairs in each exercise are for the same direct variation. Find the missing values.

13. (3,4) and (9,y)

14. (–1,2) and (4,y)

15. (3,2) and (x,3)

16. (2,y) and (-3,3)

Unit 5 SECTION 5 WORKSHEET – ANSWER SHEET

For the data in each table, tell whether y varies directly with x. If it does, write an equation for the direct variation.

1. Yes, y = 1.6x

x	y
3	4.8
5	8
8	12.8

2. yes, y= -0.25x

x	y
-2	.5
2	-.5
4	-1

3. No

x	y
6	4.5
9	6
12	9

4. No

x	y
1	3
2	2
3	1

Write an equation for the direct variation that includes the given point.

5. (2,4) y = 2x

6. (0.5, 3) y = 6x

7. (-2, $\frac{1}{8}$) y = -0.0625x

8. $(-3,5)$ y = -1.6x

9. (1.2,7.2) y = 6x

10. (4.8,3.6) y = 0.75x

11. (2.4,4.2) y = 1.75x

12. (3.4,5.1) y = 1.5x

The ordered pairs in each exercise are for the same direct variation. Find the missing values.

13. (3,4) and (9,y) y = 12

14. (–1,2) and (4,y) y = -8

15. (3,2) and (x,3) x = 4.5

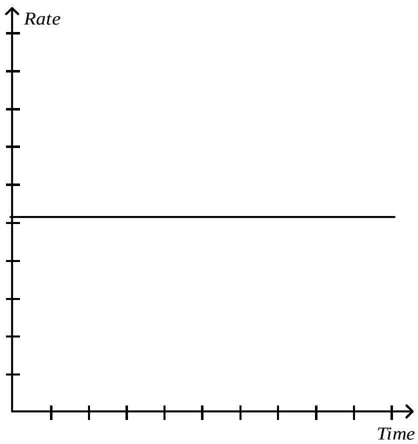
16. (2,*y*) and (-3,3) y = -2

Many graphs in this section do not have numbers along the axes. You can analyze a graph based on the shape of a graph alone.

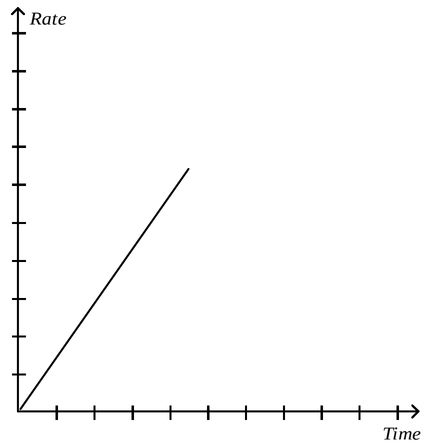
Example #3: Analyzing Graphs

The graphs below show travel speed over time.

Graph 1

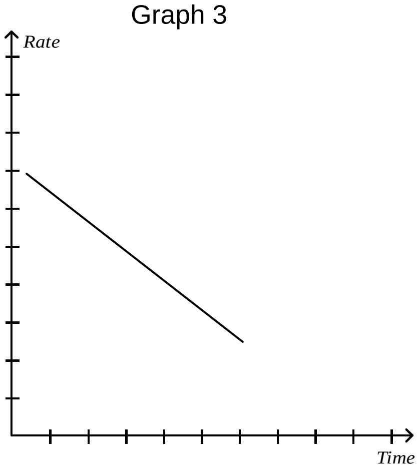


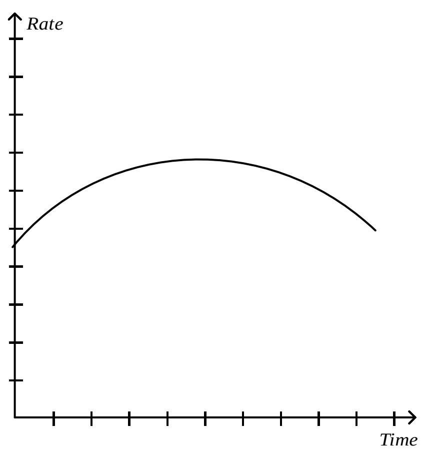
Graph 2



Graph 1 shows travel at a constant rate of speed. As the variable on the x-axis is changing, the variable on the y-axis stays the same. This shows that the variables a4re not affecting each other. Graph 2 shows a speed increase over time (acceleration). The line is straight. This means the increase occurs at a constant rate.

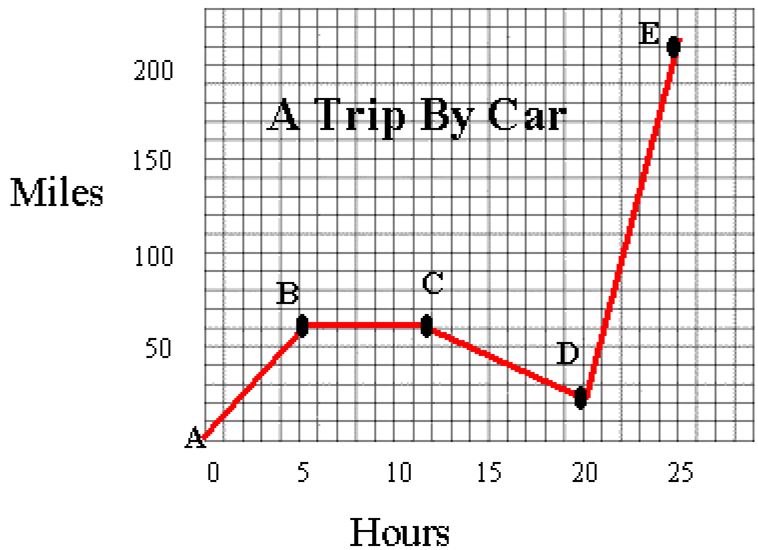
Graph 3



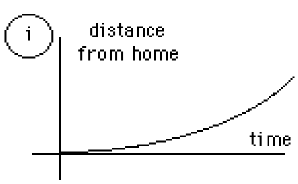


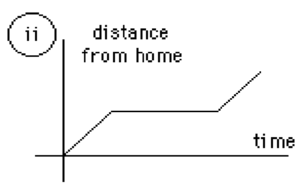
Graph 3 shows deceleration. As the variable on the x-axis increases, the variable on the y-axis decreases. The line is straight. This means the decrease occurs at a constant rate. In Graph 4, as the x-axis variable increases, the y-axis variable increases then decreases. The line is curved so the rate is changing.

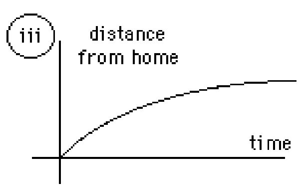
1. Answer these questions about the graph on the right:
    a. How many total miles did the car travel?
    b. What was the average speed of the car for the trip?
    c. Describe the motion of the car between hours 5 and 12?
    d. What direction is represented by line CD?
    e. How many miles were traveled in the first two hours of the trip?
    f. Which line represents the fastest speed?

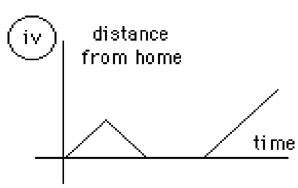


2. Identify the graph that matches each of the following stories:
    a. I had just left home when I realized I had forgotten my books so I went back to pick them up.
    b. Things went fine until I had a flat tire.
    c. I started out calmly, but sped up when I realized I was going to be late

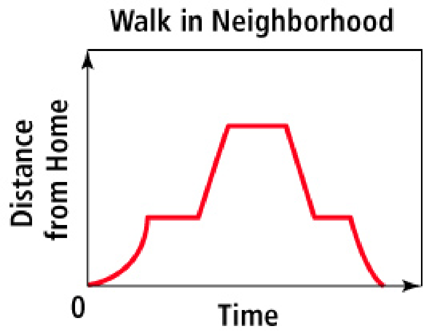


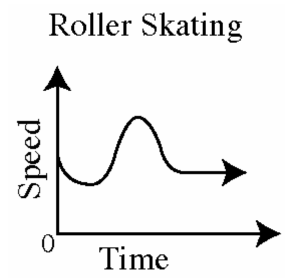






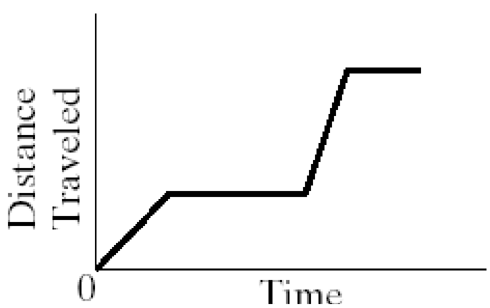
3. Copy each graph. Label each section of the graph and describe what may be happening..

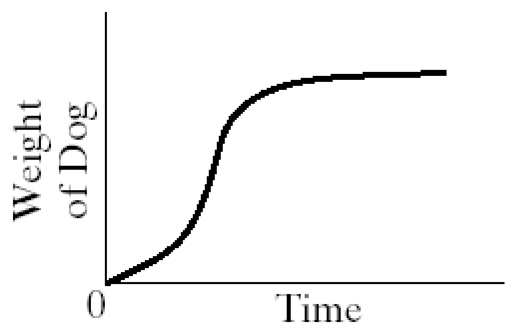




4. Sketch a graph of the daily high temperature over the course of one year for your town. A) How would the graph be different if you lived at the equator? B) How would the graph be different if you lived in Alaska?

5. Copy each graph. Label each section of the graph and describe what may be happening.

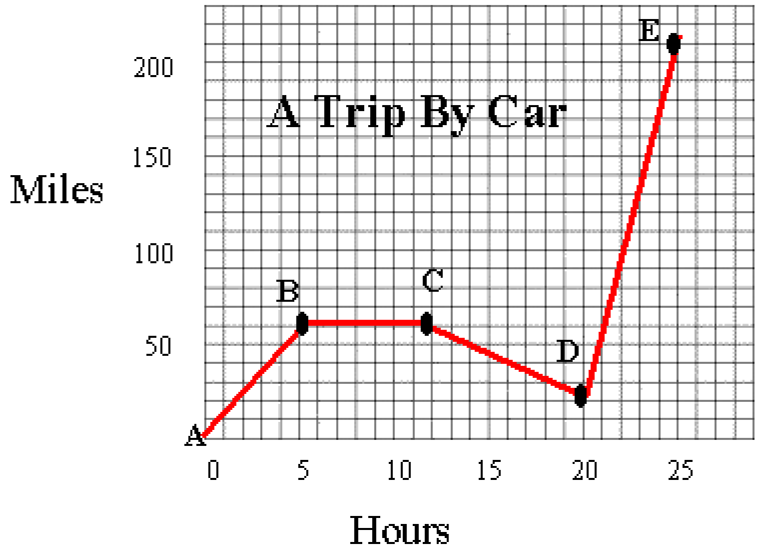
a) 

b) 

Unit 5 SECTION 1 WORKSHEET – ANSWER SHEET

1. Answer these questions about the graph on the right:

   d. How many total miles did the car travel? 210
   e. What was the average speed of the car for the trip? 8.4
   f. Describe the motion of the car between hours 5 and 12? Stopped
   g. What direction is represented by line CD? Going back towards the start
   h. How many miles were traveled in the first two hours of the trip? 25
   i. Which line represents the fastest speed? Line DF

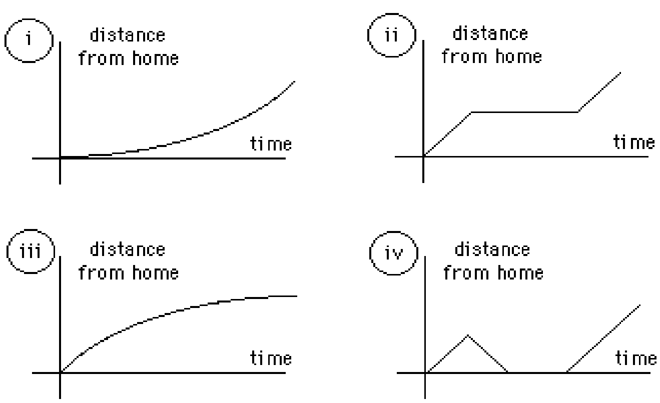


2. Identify the graph that matches each of the following stories:

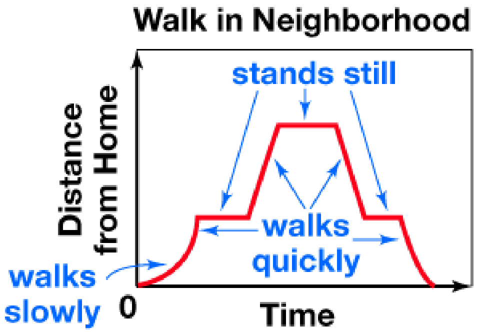
   a. I had just left home when I realized I had forgotten my books so I went back to pick them up. iv

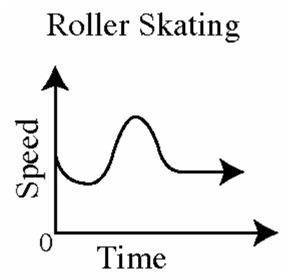
   b. Things went fine until I had a flat tire. ii

   c. I started out calmly, but sped up when I realized I was going to be late. i



3. Copy each graph. Label each section of the graph and describe what may be happening..

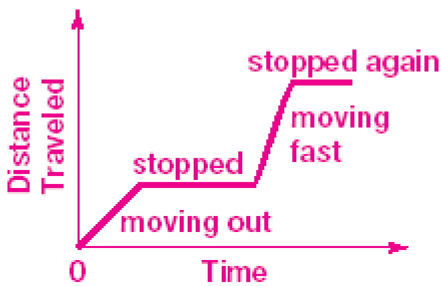




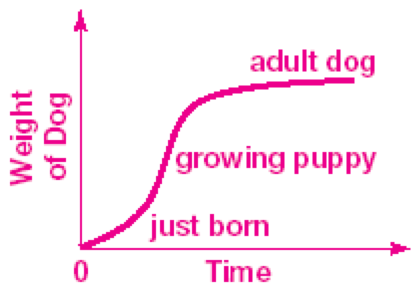
At first speed slows, then increases as is going downhill. After it peaks, the speed slows again as if going up hill and then levels out as if skating on a flat surface.

4. Sketch a graph of the daily high temperature over the course of one year for your town. A) How would the graph be different if you lived at the equator? B) How would the graph be different if you lived in Alaska? See student's work. A) A graph of temperatures at the equator would show little change for the daily high temperature. B) A graph of temperature in Alaska would show far greater changes for the daily high temperatures.

5. Copy each graph. Label each section of the graph and describe what may be happening.



a)



b)