Unit 6 – Linear Equations and Their Graphs

Section 1 – Rate of Change and Slope

Vocabulary:

Rate of Change = $\frac{\text{change in the dependent variable}}{\text{change in the independent variable}}$

Slope (m) = $\frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$

m = a letter used to indicate slope in the linear-equation form of a line.

OBJECTIVE #1: FINDING RATES OF CHANGE

Rate of change allows you to see the relationship between two quantities that are changing. If one quantity depends on the other, then the following is true:

Rate of Change = $\frac{\text{change in the dependent variable}}{\text{change in the independent variable}}$

Rate of changed is often used in our everyday lives. Parents measure a child's growth and mark it on a wall from year to year recording the rate of change. New born babies get weighed at the doctors to see that they're eating properly. Planes track their altitude over a time period to determine a rate of change called rate of descent when they're coming in for a landing. We often compare distance traveled over a time period to get a rate of change we convert to a unit rate we call miles per hour.

EXAMPLE #1: Finding rates of change using a table.

Using the data in the table, determine if the rate of change for each pair of consecutive days is the same. What does the rate of change represent?

Computer Rental

Number of Days	Rental Charge
1	80
2	100
3	120
4	140
5	160

rate of change = $\frac{\text{change in cost}}{\text{change in number of days}}$

Cost depends on the number of days.

Day 2 Change: $\frac{100\text{-}80}{2\text{-}1} = \frac{20}{1}$

Day 3 Change: $\frac{120\text{-}100}{3\text{-}2} = \frac{20}{1}$

Day 4 Change: $\frac{140\text{-}120}{4\text{-}3} = \frac{20}{1}$

Day 5 Change: $\frac{160\text{-}140}{5\text{-}4} = \frac{20}{1}$

The rate of change for each consecutive pair of days is $\frac{20}{1}$. The rate of change is the same for all data. It costs $20.00 for each day a computer is rented after the first day.

Can you calculate the rate of change using Days 5 and 2?

What's the change in cost? $160 - $100 = 60

What's the change in days? 5 – 2 = 3

Therefore, the rate of change is $\frac{60}{3} = \frac{20}{1}$

Unit 6 – Linear Equations and Their Graphs
Section 6 – Scatter Plots and Equations of Lines

Vocabulary:

Line of Best Fit – the trend line that shows the relationship between two sets of data most accurately.

Correlation – refers to the way the points are scattered on the plot. If the points are scattered randomly, there is no relationship between the sets of data and there is no correlation.

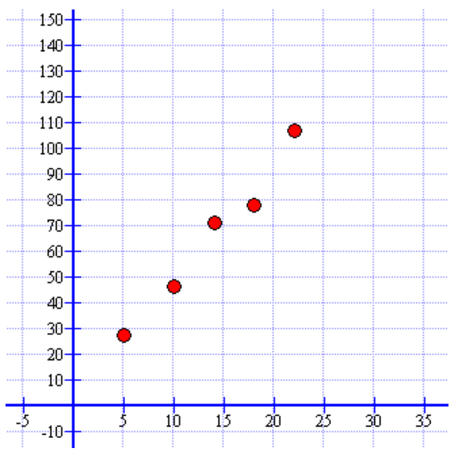
OBJECTIVE #1: Writing an Equation for a Trend Line

In Chapter 1, you worked with scatter plots, trend lines, and lines of best fit. Using what we've learned about writing linear equations, we can write equations for the trend lines of scatter plots.

EXAMPLE #1: Draw a Trend Line and Write Its Equation
Given: Data is graphed on a scatter plot. Draw the trend line and write its equation. Estimate the time for a 32 miles trip.

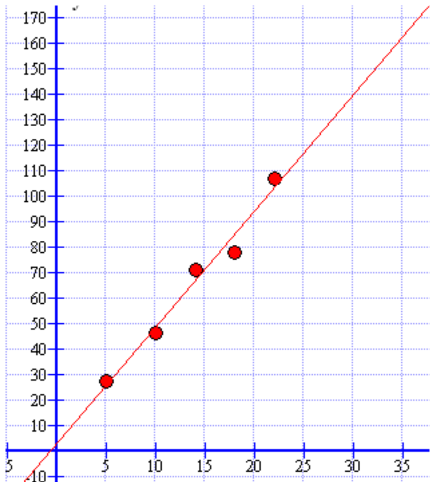
Step One: Graph the data.

This data represents miles traveled on the x-axis and time in minutes on the y-axis.



Step Two: Draw a trend line.

If the information has a good correlation, there should be about as many data points above the trend line as there are below the trend line.



Step Three: Write an equation for the trend line.

Select two points on the trend line and use their coordinate values to write an equation for the trend line. The points selected are (5, 25) and (30, 140).

A) Calculate the slope:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{140 - 25}{30 - 5} = \frac{115}{25} = \frac{23}{5}$$

B) Use the Point-Slope form
(5,25) for (x_1, y_1)

$y - 25 = \frac{23}{5}(x - 5)$ Substitute the slope for m and (5,25) for (x_1, y_1)

Step Four: Estimate the amount of time for a 32 mile bicycle trip.

$y - 25 = \frac{23}{5}(x - 5)$	Use the equation we just wrote.
$y - 25 = \frac{23}{5}(32 - 5)$	Substitute 32 for the x value.
$y - 25 = \frac{23}{5}(27)$	Simplify within the parentheses
$y - 25 = 124.2$	Multiply
$y = 149.2$	Add 25 to both sides

A 32 mile bike ride would take about 149 minutes.

PRACTICE – AMSCO ALGEBRA 1 BOOK
Do problems #1-13 (All) on Page 618

Unit 6 – Linear Equations and Their Graphs

Section 5 – Parallel and Perpendicular Lines

Vocabulary:

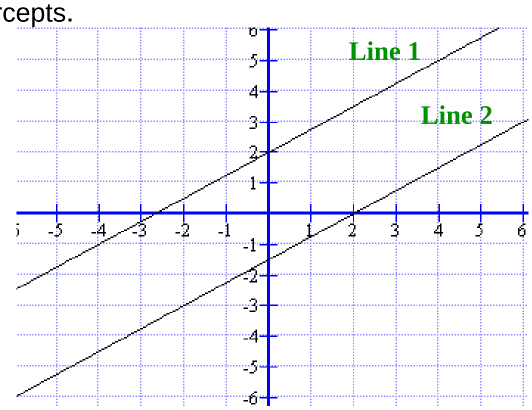
Parallel Lines – lines in the same plane that never intersect

Perpendicular Lines – lines that intersect to form right angles

Negative Reciprocal – the product of a number and its negative reciprocal is -1

OBJECTIVE #1: Parallel Lines

In the graph below we see two lines that are parallel. Parallel lines are lines in the same plane that never meet. What you will notice about the lines is that they have the same slope but have different y-intercepts.



Non-vertical lines are parallel if they have the same slope but different y-intercepts. Any two vertical lines are parallel.

Line 1 above represents the equation $y = \frac{3}{4}x + 2$

Line 2 above represents the equation $y = \frac{3}{4}x - 1\frac{1}{2}$

Both lines have the same slope but different y-intercepts.

EXAMPLE #1: Determining Whether Lines Are Parallel

Given: Given the equations for two lines, determine whether the lines are parallel.

Line 1 - $y = -\frac{1}{3}x + 5$

Line 2 - $2x + 6y = 12$

Step One: Write the standard from of line 2 in the slope-intercept form of line so we can compare them.

$2x + 6y = 12$

$6y = -2x + 12$ Subtract 2x from each side

$\frac{6y}{6} = \frac{-2x + 12}{6}$ Divide both sides by 6

$y = -\frac{1}{3} + 2$ Simplify

The lines are parallel. Both lines have the slope $-\frac{1}{3}$ and different y-intercepts.

EXAMPLE #2: Writing Equations of Parallel lines

You can use the fact that the slopes of parallel lines are the same to write the equation of a line parallel to a given line. To write the equation, you use the slope of the given line and the point-slope form of a linear equation.

Given: Write an equation for a line that is parallel to $y = \frac{2}{3}x + 4$ and passes through (3, 1)

Step One: Identify the slope of the given line = $\frac{2}{3}$

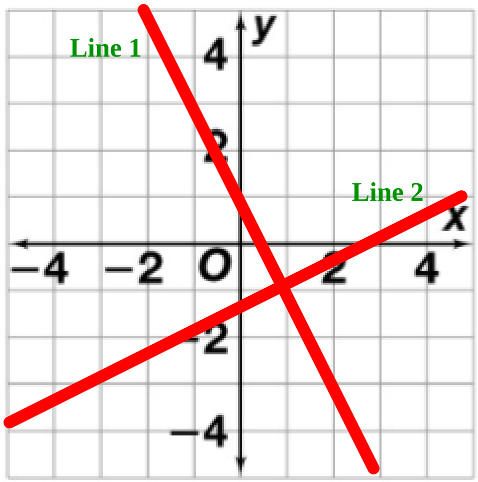
Step Two: Write an equation for the line passing through (3, 1) using the slope-intercept form.

$y - y_1 = m(x - x_1)$	Point-slope Form
$y - 1 = \frac{2}{3}(x - 3)$	Substitute (3, 1) for (x_1, y_1) and $\frac{2}{3}$ for m
$y - 1 = \frac{2}{3}x - \frac{2}{3}(3)$	Use the Distributive Property
$y - 1 = \frac{2}{3}x - 2$	Simplify
$y = \frac{2}{3}x - 1$	Add 1 to both sides

The line represented by $y = \frac{2}{3}x - 1$ is parallel to $y = \frac{2}{3}x + 4$ and passes through (3,1)

OBJECTIVE #2: Perpendicular Lines

In the graph below, we see two lines that are perpendicular to each other. Perpendicular lines are lines that intersect to form right angles. Their slopes are <u>negative reciprocals</u> of each other; that is to say, if you multiplied the two slopes you would get -1 as the product.



The equation for Line 1 is $y = 2x + 1$ while the equation for Line 2 is $y = -\frac{1}{2}x - 1\frac{1}{2}$. The slope of Line 1 =2. The slope of Line 2 = $-\frac{1}{2}$. They are negative reciprocals of each other. $2 \bullet -\frac{1}{2} = -1$

You can use the negative reciprocal of the slope of a given line to write an equation of a line perpendicular to that line.

EXAMPLE #3: Writing Equations for Perpendicular Lines

Given: Write the equation of the line that contains (0, -4) and is perpendicular to $y = 4x + 1$

Step One: Identify the slope of the given line $y = 4x + 1$

^ slope = 4

Step Two: Find the negative reciprocal of the slope

The negative reciprocal of 4 is $-\frac{1}{4}$

Step Three: Use the slope-intercept form to write an equation.

$y = mx + b$

$y = -\frac{1}{4}x + (-4)$ Substitute $-\frac{1}{4}$ for m, and -4 for b

$y = -\frac{1}{4}x - 4$ Simplify

Unit 6 Section 5 Worksheet

Write an equation for the line that is parallel to the given line and passes through the given point.

1. $y = 5x - 1$; (0,0)
2. $y = \frac{5}{2}x + 6$; (-4,-6)
3. $y = \frac{2}{3}x - 1$; (3,0)
4. $y = .5x + 6$; (4,2)

Find the slope of a line that is perpendicular to the line represented by the equation.

5. $y = 3x$
6. $y = -\frac{3}{4}x + 2$
7. $3y + 2x = 6$
8. $y = \frac{4}{7}x - 7$

Write an equation for the line that is perpendicular to the line represented by the equation and passes through the given point.

9. $y = \frac{1}{4}x + 3$; (0,0)
10. $2x + 4y = 6$; (1,4)
11. $y = x - 4$; (2,3)
12. $-4x - 2y = 8$; (5,4)

13. Graph the line that passes through (3, 5) and is parallel to y= 2x + 1

14. Graph the line that passes through (4, 2) and is perpendicular to 2x + 3y = 4

Unit 6 Section 5 Worksheet – ANSWER SHEET

Write an equation for the line that is parallel to the given line and passes through the given point.

1. $y = 5x - 1$; (0,0) y = 5x

2. $y = \frac{5}{2}x + 6$; (-4,-6) $y = \frac{5}{2}x + 14$

3. $y = \frac{2}{3}x - 1$; (3,0) $y = \frac{2}{3}x - 2$

4. $y = .5x + 6$; (4,2) y = .5x

Find the slope of a line that is perpendicular to the line represented by the equation.

5. $y = 3x$ -1/3

6. $y = -\frac{3}{4}x + 2$ 4/3

7. $3y + 2x = 6$ 3/2

8. $y = \frac{4}{7}x - 7$ -7/4

Write an equation for the line that is perpendicular to the line represented by the equation and passes through the given point.

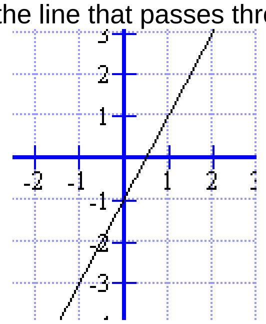
9. $y = \frac{1}{4}x + 3$; (0,0)
y = -4x

10. $2x + 4y = 6$; (1,4) y = 2x + 2

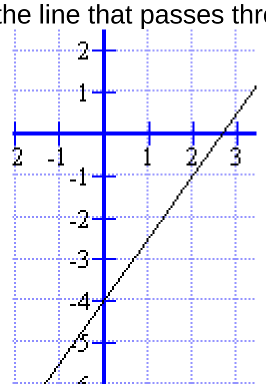
11. $y = x - 4$; (2,3)
y = -x + 5

12. $-4x - 2y = 8$; (5,4) y = .5x + 1.5

13. Graph the line that passes through (3, 5) and is parallel to y= 2x + 1



14. Graph the line that passes through (4, 2) and is perpendicular to 2x + 3y = 4



Unit 6 – Linear Equations and Their Graphs
Section 4 – Point-Slope Form and Writing Equations

Vocabulary:

Point-Slope Form of a Linear Equation – the equation of a non-vertical line that passes through the point (x_1, y_1) with slope m is $y - y_1 = m(x - x_1)$

OBJECTIVE #1: Using Point-Slope Form

If you know a line passes through a particular point and has a particular slope, you can use the Point-Slope form of an equation to quickly write the equation of the line

EXAMPLE #1: Using Point-Slope Form to Write the Equation of a Line

Given: You know a line passes through the point (3, 5) with a slope =2.

$\frac{2}{3}$ Use the Point-Slope Form of a Linear Equation

$y - 5 = m(x - 3)$ Substitute (3, 5) for (x_1, y_1)

$y - 5 = 2(x - 3)$ Substitute the value 2 for m

The equation $y - 5 = 2(x - 3)$ is the point-slope form of the equation of the line.

NOTE: If this seems too easy, we can do some math and put this in the slope-intercept form of an equation. Seems easier to just plug the values into the Point-Slope Form.

$y - 5 = 2(x - 3)$

$y - 5 = 2x - 6$ Use the Distributive Property

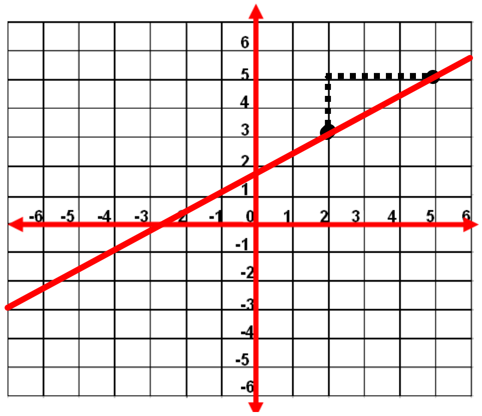
$y = 2x - 1$ Add 5 to both sides

EXAMPLE #2: Graphing Using Point-Slope Form

Given: Graph the equation $y - 3 = \frac{2}{3}(x - 2)$

$y - 3 = \frac{2}{3}(x - 2)$ The equation is in point-slope form so we know the slope = $\frac{2}{3}$ and we know the line passes through (2, 3).

Step One: Plot point (2, 3) on the graph.
Step Two: Use the slope to plot another point on the graph then draw the line through the two points.



If you know two points on a line, first use them to find the slope, then use either point to write the equation for the line.

EXAMPLE #3: Using Two Points to Write an Equation

Given: Write equations for the line passing through (-1, 4) and (2, 3) in both point-slope for, and in slope intercept form.

Step One: Calculate the slope of the line

Slope (m) = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{3-4}{2-(-1)} = \frac{-1}{3} = -\frac{1}{3}$

Step Two: Select either point to write the equation in point-slope form. For this example, we'll use (2, 3)

$y - y_1 = m(x - x_1)$ Point-Slope Form

$y = -\frac{1}{3}x + 3\frac{2}{3}$ Substitute (2, 3) for (x_1, y_1) and $-\frac{1}{3}$ for m

Step Three: Rewrite the equation from Step Two in slope-intercept form.

$y - 3 = -\frac{1}{3}(x-2)$

$y - 3 = -\frac{1}{3}x + \frac{2}{3}$ Use the Distributive Property

$y = -\frac{1}{3}x + 3\frac{2}{3}$ Add 3 to both sides

OBJECTIVE #2: Writing an Equation Using Data

Linear equations can be written from data in tables when the rate of change between consecutive pairs of data is the same. If the data has a linear relationship, then the rate of change is the slope.

EXAMPLE #4: Writing an Equation Using a Table

Given: You are given a table. Determine if the data is linear. If the data is linear, write an equation that models the data.

Stage	x	y
1	-2	3
2	2	5
3	4	6
4	10	9

Step One: Find the rate of change for the consecutive ordered pairs

From Stage 1 to Stage 2 : $\frac{5-3}{2-(-2)} = \frac{2}{4} = \frac{1}{2}$

From Stage 2 to Stage 3: $\frac{6-5}{4-2} = \frac{1}{2}$

From Stage 3 to Stage 4: $\frac{9-6}{10-4} = \frac{3}{6} = \frac{1}{2}$

The rate of change is constant and the slope of the line = $\frac{1}{2}$

Step Two: Use the slope and a point to write an equation

The slope = $\frac{1}{2}$ and we can use any one of the four pairs of data in the table. For this exercise let's pick (4,6).

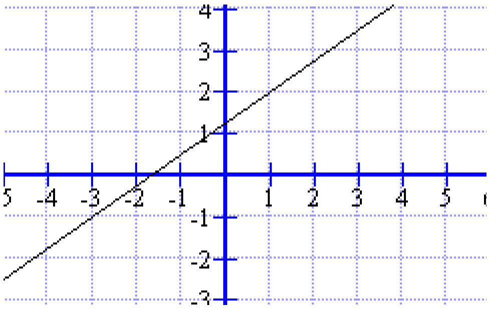
$y - y_1 = m(x - x_1)$	Point-slope form of linear equation
$y - 6 = \frac{1}{2}(x - 4)$	Substitute $\frac{1}{2}$ for m and (4,6) for (x_1, y_1)

The equation for the line representing the data in the table is $y - 6 = \frac{1}{2}(x - 4)$

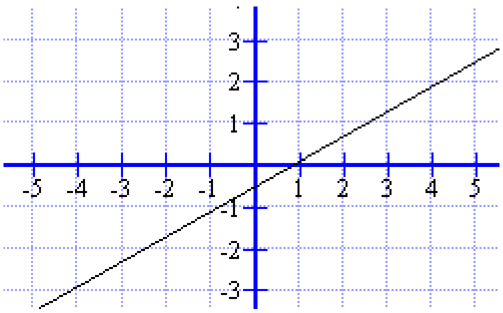
Unit 6 Section 4 Worksheet

Write an equation of each line in point-slope form:

1.



2.



Write one equation of the line passing through the given points in point-slope form and then write one in standard form using integers (no fractions):

3. (2, 5), (-1, 3)
4. (6, -1), (1, 2)
5. (3, 5), (-2, 4)
6. (2, 4), (-4, -8)
7. (-5, -5), (2, 2)
8. (7, -3), (-2, -3)

Is the data in the table linear? If so, model the data with an equation.

9.

x	y
-5	8
1	-4
4	-10
8	-18
13	-28

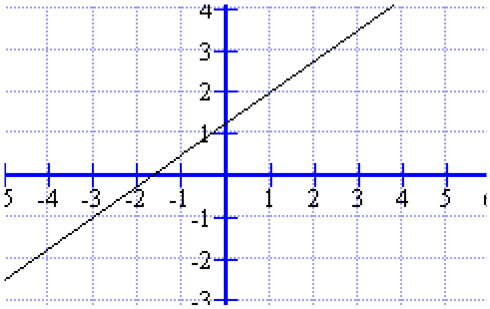
10.

Overdue Book Fines	Amount Of Fine
3 days	$1.75
5 days	$2.25
9 days	$3.25
20 days	$6.00

Unit 6 Section 4 Worksheet – ANSWER SHEET

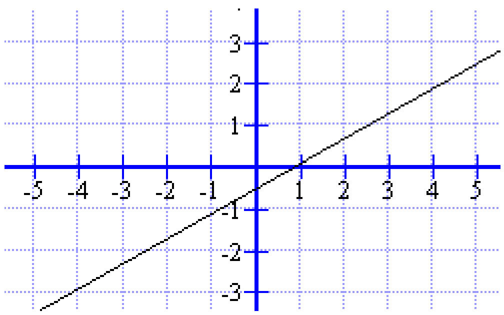
Write an equation of each line in point-slope form:

1.



Answers may vary $y - 2 = \frac{3}{4}(x - 1)$

2.



Answers may vary $y + 3 = \frac{3}{5}(x + 4)$

Write one equation of the line passing through the given points in point-slope form and then write one in standard form using integers (no fractions):

9. (2, 5), (-1, 3)

   $y - 5 = \frac{2}{3}(x - 2)$;5y= 3x - 3

10. (6, -1), (1, 2)

    $y - 2 = -\frac{3}{5}(x - 1)$;5y = -3x + 13

11. (3, 5), (-2, 4) $y - 4 = \frac{1}{5}(x + 2)$

    5y = x + 22

12. (2, 4), (-4, -8) y – 4 = 2 (x – 2)
    y = 2x

13. (-5, -5), (2, 2) y - 2 = x – 2
    y = x

14. (7, -3), (-2, -3) y + 3 = 0
    y = -3

Is the data in the table linear? If so, model the data with an equation.

9. Yes; answers may vary
   y – 8 = -2(x + 5)
   y = -2x - 2

x	y
-5	8
1	-4
4	-10
8	-18
13	-28

10. Yes; answers may vary

$y - 1.75 = .25(x - 3)$

$4y - 7 = x - 3$

$y = .25x + 1$

Overdue Book Fines	Amount Of Fine
3 days	$1.75
5 days	$2.25
9 days	$3.25
20 days	$6.00

Unit 6 – Linear Equations and Their Graphs
Section 3 – Standard Form

Vocabulary:

Standard Form of a Linear Equation – a linear equation expressed in the form $x=2$ where A, B, and C are real numbers and A and B are not both zero.

x-intercept – the coordinate value of x when a line crosses the x-axis.

y-intercept – the coordinate value of y when a line crosses the y-axis.

OBJECTIVE #1: Graphing Equations Using Intercepts

You are familiar with the slope-intercept form of a linear equation, but that is just one form of a linear equation. Another form of a linear equation is standard from. Standard form is useful for making quick graphs using the x-intercept and y-intercept. To graph a linear equation in standard from, you can find the x-intercept by substituting 0 for the y-value and solving for x. Similarly, you can find the y-intercept by substituting 0 for the x-value and solving for y.

EXAMPLE #1: Finding x- and y-intercepts.

Given: A linear equation in standard from. $4x+3y=8$

First Step: to find the x-intercept, substitute 0 for the y-value and solve the equation for x.

$$4x+3y=8$$
$$4x+3(0)=8$$
$$4x=8$$
$$x=2$$

The x-intercept is 2.

Second Step: to find the y-interceptm substitute 0 for the x-value and solve for y.

$$4x+3y=8$$
$$4(0)+3y=8$$
$$3y=8$$
$$y=\frac{8}{3}$$

The y-intercept is $\frac{8}{3}$.

EXAMPLE #2: Graphing Lines Using Intercepts

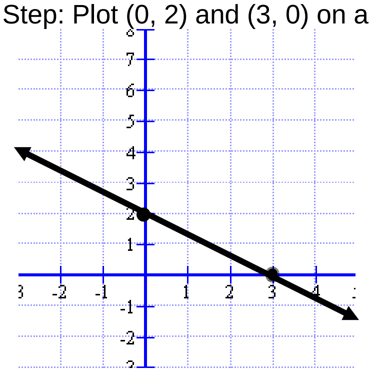
Given: A linear equation in standard from $4x+6y=12$

First Step: find both the x-intercept and the y-intercept.

$4x+6y=12$	$4x+6y=12$
$4x+6(0)=12$	$4(0)+6y=12$
$4x=12$	$6y=12$
$x=3$	$y=2$

Now we know that when y=0, x= 3, and when x=0, y=2

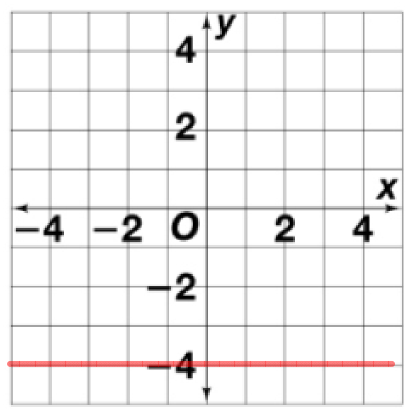
Second Step: Plot (0, 2) and (3, 0) on a graph and draw a line through the points.



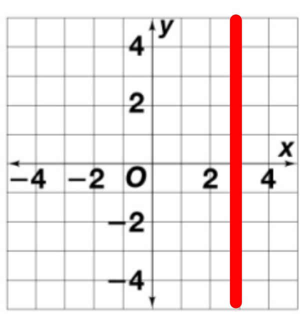
EXAMPLE #3: Graphing Horizontal and Vertical Lines

In the definition of standard form of a linear equation, it was stated that A and B are not <u>both</u> zero. If they were both zero, then we would not have a line. However, one or the other of them can be zero. When that happens, we have either a horizontal or a vertical line, depending on which factor is zero.

Given: $y = -4$ If written in standard from we have $0x + 1y = -4$ and for all values of x, y = -4. This gives us a horizontal line.



Given: $x = 3$ If written in standard from, we have $1x + 0y = 3$ and for all values of y, x = 3. This gives us a vertical line.



OBJECTIVE #2: Writing Equations in Standard Form

You can change a linear equation from slope-intercept form to standard from. If the slope-intercept form contains fractions or decimals, multiply to write the equation using integers.

EXAMPLE #4: Transforming Slope-Intercept Form to Standard form.

Given: A linear equation in slope-intercept form to be transformed into standard form.

$$y = \frac{3}{5}x + 2$$

First Step: Multiply both sides by 5	$5y = 5(\frac{3}{5}x + 2)$
Second Step: Use the Distributive Property	$5y = 3x + 10$
Third Step: Subtract 3x from both sides	$-3x + 5y = 10$

The standard form of $y = \frac{3}{5}x + 2$ is $-3x + 5y = 10$.

EXAMPLE #5: Real-World Problem Solving

A delivery service has trucks that run on either gasoline or ethanol. Gasoline costs $3.00 per gallon and ethanol only costs $2.25 per gallon, but ethanol is not available in all locations. Write an equation in standard from to relate the number of gallons of ethanol and the number of gallons of gasoline a driver may purchase if he has a weekly fuel allowance of $400.

Define: x = the number of gallons of gasoline at a cost of $3.00 per gallon.
y = the number of gallons of ethanol at a cost of $2.25 per gallon.

Write: $3x + 2.25y = 400$

Unit 6 Section 3 Worksheet

Graph each equation using x- and y-intercepts;

1) $x + y = 3$

2) $x + y = -2$

3) $x - y = 4$

4) $-2x + y = -5$

For each equation, tell whether it is horizontal or vertical:

5) $y = -3$

6) $x = 2$

7) $y = 1.5$

8) $x = -4.1$

Write each equation in standard form using integers (no fractions):

9) $y = 4x + 1$

10) $y = \frac{1}{2}x - 3$

11) $y = -\frac{2}{3}x + 2$

12) $y = \frac{3}{4}x + \frac{1}{2}$

Graph each equation:

13) $x + y = 2$

14) $y = -3x - 3$

15) $y - x = -1$

16) $2x - 3y = 4$

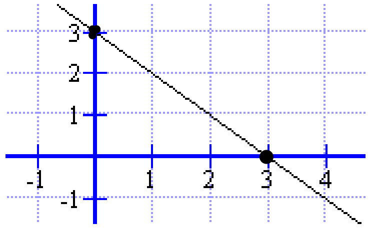
17) $3 - y = x - 2$

18) $6 + y = 5 - x$

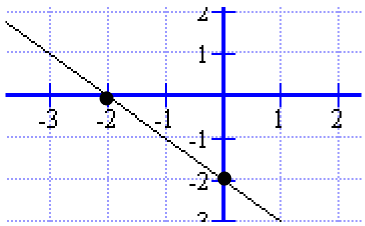
Unit 6 Section 3 Worksheet – ANSWER SHEET

Graph each equation using x- and y-intercepts;

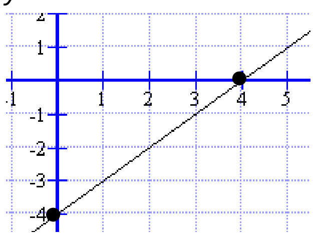
1) $x+y=3$



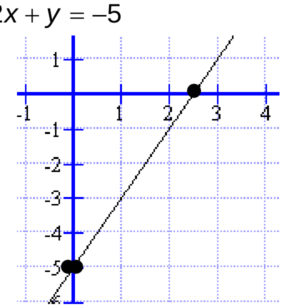
2) $x+y=-2$



3) $x-y=4$



4) $-2x+y=-5$



For each equation, tell whether it is horizontal or vertical:

5) $y=-3$ Horizontal

6) $x=2$ Vertical

7) $y=1.5$ Horizontal

8) $x=-4.1$ Vertical

Write each equation in standard form using integers (no fractions):

9) $y=4x+1$ -4x + y = 1

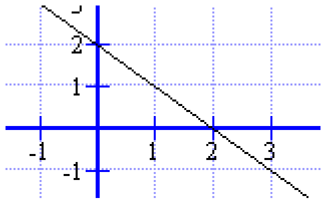
10) $y=\frac{1}{2}x-3$ -x + 2y = -6

11) $y=-\frac{2}{3}x+2$ 2x + 3y = 6

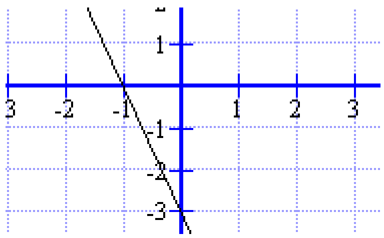
12) $y=\frac{3}{4}x+\frac{1}{2}$ -3x + 4y = 2

Graph each equation:

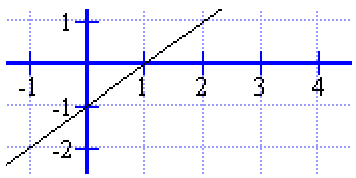
13) $x + y = 2$



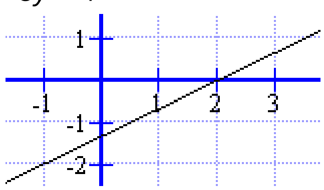
14) $y = -3x - 3$



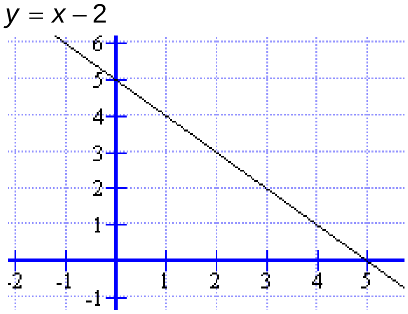
15) $y - x = -1$



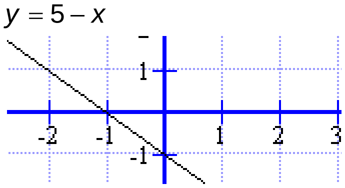
16) $2x - 3y = 4$



17) $3 - y = x - 2$



18) $6 + y = 5 - x$



Unit 6 – Linear Equations and Their Graphs
Section 2 – Slope-Intercept Form

Vocabulary:

<u>Linear Equation</u> – an equation whose graph is a straight line.

<u>y-intercept</u> – the value of y when the x-value is zero (0) or the y-coordinate when the line crosses the y-axis. Represented by the letter "b" when a linear equation is expressed in the slope-intercept form.

<u>Slope-Intercept Form</u> – a linear equation is in slope-intercept form when it is expressed in the form $y=mx+b$ where m = the slope of the line and b = the y-intercept.

OBJECTIVE #1: WRITING LINEAR EQUATIONS

To write an equation for a line, determine its slope and y-intercept. Use the slope-intercept form for the equation (y = mx + b) and substitute the slope value for "m" and the y-intercept value for "b".

If you know the slope of a line and any coordinate pair on the line, you can begin with the slope-intercept form of a line and substitute the slope value for "m", the plug in the coordinate values for the 'x' and 'y' and determine the y-intercept value. Substitute it for 'b' in the slope-intercept form and you have the equation of the line.

EXAMPLE #1:Write the equation for a line given a slope of 3 and a y-intercept of 4.

First, begin with the slope-intercept form of a linear equation.	y = mx + b
Substitute the slope of 3 for 'm'	y = 3x + b
Substitute the y-intercept value of 4 for 'b'	y = 3x + 4

The equation y = 3x + 4 represents the line and the coordinates of any point on the line will satisfy this equation. If you are given a coordinate pair and the (x,y) values do not satisfy the equation, that coordinate pair does not lie on the line.

EXAMPLE #2: Write the equation for a line given the slope and a coordinate pair on the line.

Given: A line with a slope of -2 and the coordinate pair (3, 1) which lies on the line.

First, begin with the y-intercept form of the equation	y = mx + b
Substitute -2 for the 'm' value.	y = -2x + b
Substitute 3 and 1 for the x and y values	1 = -2(3) + b
Solve for 'b' to find the y-intercept	1 = -6 + b b = 7
Now substitute the 7 for the 'b' value using the y-intercept form	y = -2x + 7

The equation y = -2x + 7 represents the line and the coordinates of all points on the line.

EXAMPLE #3: Write the equation for a line given two points on the line.

Given: Write the equation for the line given coordinate pairs (5, 6) and (1, -2).

1) First, start with the slope formula to find the value of 'm' $m=\frac{y_2 - y_1}{x_2 - x_1}$

2) Substitute (5, 6) for the x_2, y_2 values and (1, -2) for the x_1, y_1 values. $m=\frac{6-(-2)}{5-1}$

m = 2

3) Substitute 2 for 'm' in the y-intercept form of a linear equation. y = 2x + b

4) Pick one of the two points and substitute the x and y values of the point for the x and y in the equation. Solve for 'b'.

6 = 2(5) + b
b = 6 – 10
b = -4

5) Substitute -4 for the 'b' value in the slope-intercept form of the equation.

y = 2x + (-4) or
y = 2x -4

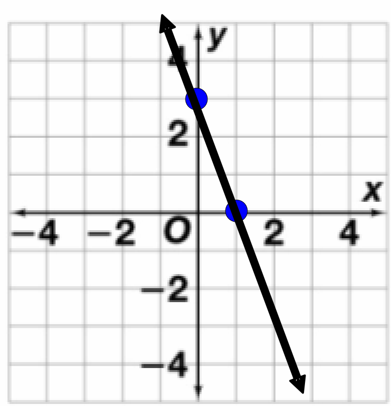
The equation y = 2x -4 represents the line and all points on the line will satisfy the equation. Any point that does not satisfy the equation does not lie on the line.

EXAMPLE #4: Writing an Equation from a Graph

Given: Write the equation of a line represented on a graph.

1) Find the slope using two points on the graph.
2) (0, 3) and (1, 0) lie on the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad m = \frac{3-0}{0-1} = \frac{3}{-1} = -3$$



3) Write an equation in slope-intercept form. We already know the y-intercept is 3 because that is the value of y when x equals zero (0).

y = -3x +3

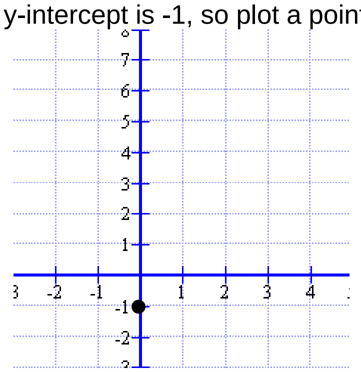
OBJECTIVE #2: Graphing Linear Equations

Each point on a graph of an equation is an ordered pair that makes the equation true. The graph of a linear equation is line that indicates all the solutions of the equation. An ordered pair on the line will satisfy the equation. An ordered pair that is not on the line will not satisfy the equation.

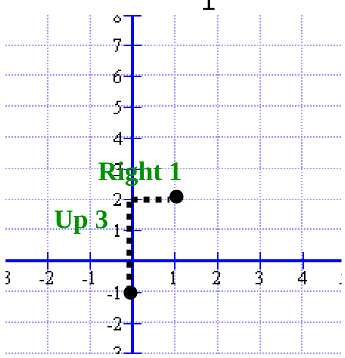
EXAMPLE #5: Graphing Equations Using Linear Equation Form

Given: The slope-intercept form of a linear equation. Example: $y = 3x - 1$

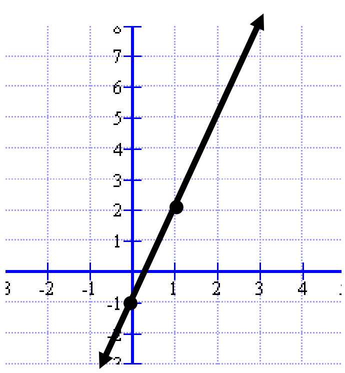
First Step: The y-intercept is -1, so plot a point at (0, -1).



Second Step: The slope is 3, or $\frac{3}{1}$. Use the slope to plot a second point.



Third Step: Draw a line through the two points.



All points on the line will satisfy the equation.

Unit 6 Section 2 Worksheet

Find the slope and y-intercept of each equation.

1. $y-1=-2x$
2. $y=5x+4$
3. $y-\frac{3}{4}x=1$
4. $-2y=4(3-x)$
5. $2y-4=6x$
6. $y-5x=\frac{2}{3}$

Use the slope and y-intercept to graph each equation.

7. $3y+6x=0$
8. $y=\frac{1}{2}x-2$
9. $y=5-2x$
10. $y-3=4x-2$
11. $3x+4y=x-2$
12. $-2(x-3)+y=0$

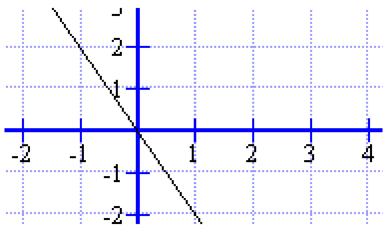
Unit 6 Section 2 Worksheet – ANSWER SHEET

Find the slope and y-intercept of each equation.

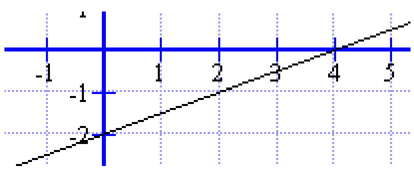
1. $y - 1 = -2x$ -2; 1
2. $y = 5x + 4$ 5; 4
3. $y - \frac{3}{4}x = 1$ 3/4; 1
4. $-2y = 4(3 - x)$ 2; -6
5. $2y - 4 = 6x$ 3; 2
6. $y - 5x = \frac{2}{3}$ 5; 2/3

Use the slope and y-intercept to graph each equation.

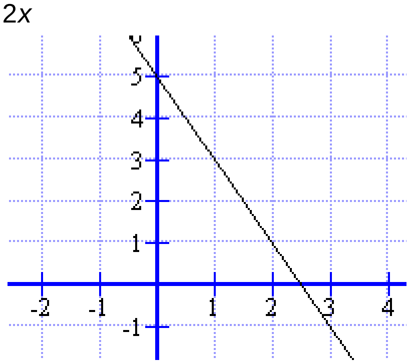
7. $3y + 6x = 0$



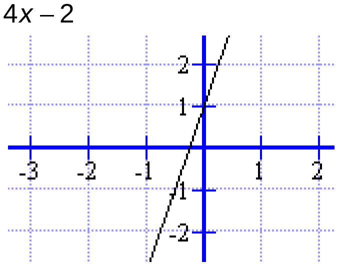
8. $y = \frac{1}{2}x - 2$



9. $y = 5 - 2x$

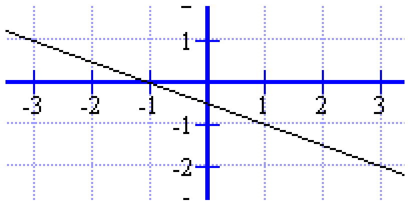


10. $y - 3 = 4x - 2$

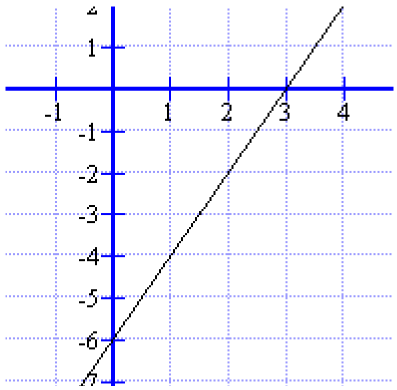


Unit 6 Section 2 Worksheet – ANSWER SHEET

11. $3x+4y=x-2$



12. $-2(x-3)+y=0$



Unit 6 – Linear Equations and Their Graphs
Section 7 – Graphing Translations of Absolute Value Equations

Vocabulary:

Absolute Value Equation – makes a V-shaped graph that points upwards or downwards.

Translation – a shift (or slide) of a graph either horizontally, vertically, or both.

OBJECTIVE #1: Translating Graphs of Absolute Value Equations

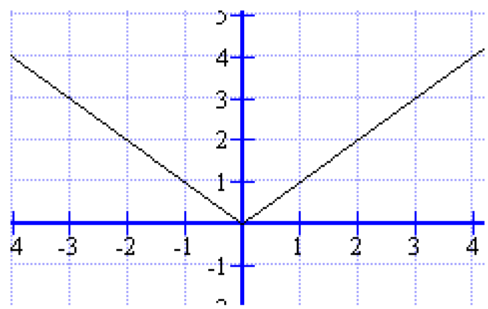
We already know the graph of an absolute value equation makes a V-shaped graph that points upwards or downwards. Previously we graphed absolute value equations by making tables of values (Chapter 5, Lesson 3).

In this lesson, we are going to work with absolute value equations and their translations. A translation is a shift in the graph, either horizontally, vertically, or both. You will learn how a change in an absolute value equation affects the resulting graph of the equation.

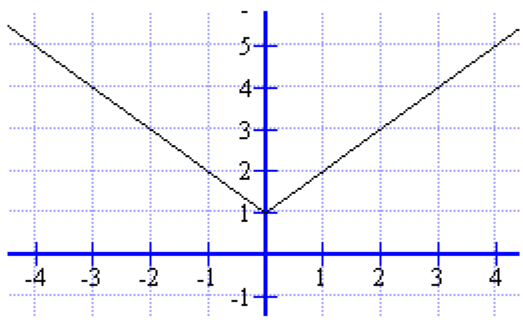
EXAMPLE #1: Vertical Translations

First, we are going to make a table and then graph the equation $y = |x|$.

x	Y
-2	2
-1	1
0	0
1	1
2	2



Now, we're going to make a graph of the equation $y = |x| + 1$



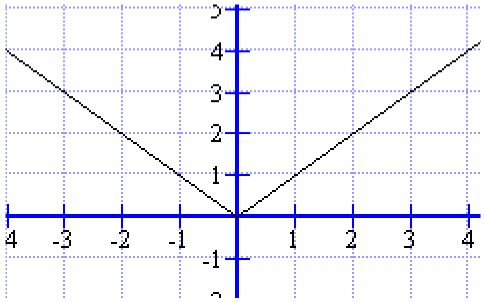
Notice the difference in the graphs. Both are the same shape. The shape did not change.

The difference is the change in the y-intercepts! $y = |x|$ has an intercept of 0, while $y = |x| + 1$ has a y-intercept of 1. y=-|x+3|

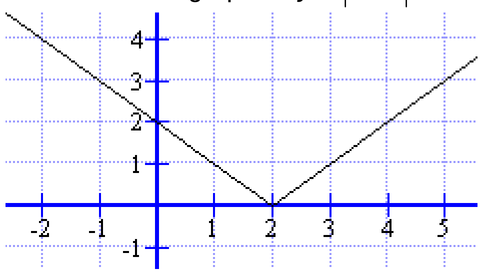
CONCLUSION: Vertical translations are affected by a number outside the absolute value sign.

EXAMPLE #2: Horizontal Translations

Now we're going to look at some graphs that demonstrate a horizontal translation. The first will be the graph of $y = |x|$ that we have seen before. The y-intercept is 0.

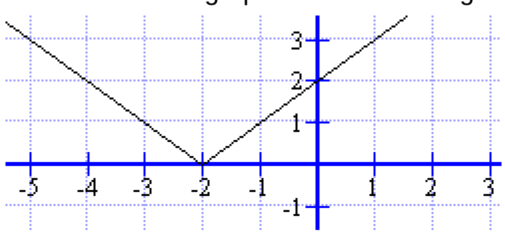


Next we will look at the graph of $y = |x - 2|$. Notice the difference in the equations.



Notice the y-intercept is now 2 and the vertex has shifted to the right two units. This makes sense because when x = 0, y will equal the absolute value of -2. When y = 0, we have $0 = |x - 2|$, which would mean x = 2.

Now, we'll look at a third graph where we change the equation slightly $y = |x + 2|$



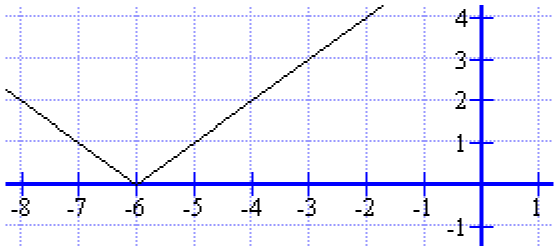
Notice the y-intercept is 2 but the vertex has moved to -2!!! This makes sense because when x = 0, y will equal the absolute value of +2. When y = 0, we have $0 = |x + 2|$, which would mean x has to equal -2.

CONCLUSION: The vertex moves in the opposite direction of what we + or – from x inside the absolute value signs.

EXAMPLE #3: Writing the Translation of an Absolute Value Equation

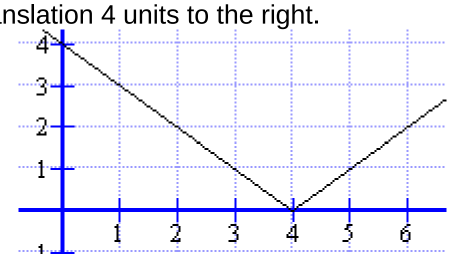
Given: Given $y = |x|$, write an equation for

a) translation 6 units to the left.



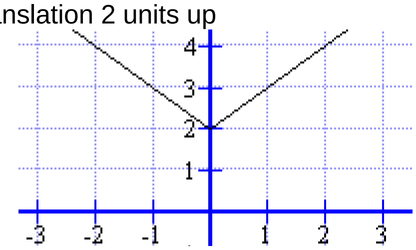
The equation representing the graph's new location would be $y = |x+6|$

b) translation 4 units to the right.



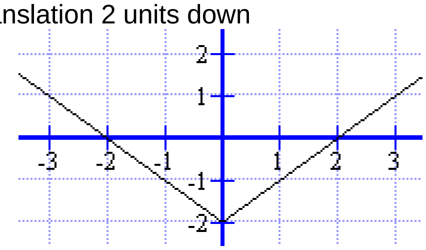
The equation representing the new location would be $y = |x-4|$.

c) translation 2 units up



The equation would be $y = |x|+2$.

d) translation 2 units down



The equation would be $y = |x|-2$

KEY POINTS: (1) Up or down translations of absolute value equations are affected by what is done <u>outside</u> the absolute value sign. (2) Left or right translations are affected by adding or subtracting <u>inside</u> the absolute value sign.

Unit 6 Section 7 Worksheet

Write an equation for each translation of $y=|x|$.

1) 7 units up
2) 4 units down
3) 3.5 units up
4) 2 units down
5) 3 units to the right
6) 1 unit to the left

Graph each function by translating $y=|x|$.

7) $y=|x|+5$
8) $y=|x|-3$
9) $y=|x|+1.5$
10) $y=|x+1.5|$
11) $y=|x+3|$
12) $y=|x-4|$

Write an equation for each translation of $y=-|x|$.

13) 3.5 units up
14) 5 units down
15) 2 units to the right
16) 3 units to the left

Graph each function by translating $y=-|x|$.

17) $y=-|x+2|$
18) $y=-|x|+2.5$
19) $y=-|x|-2$
20) $y=-|x-2|$

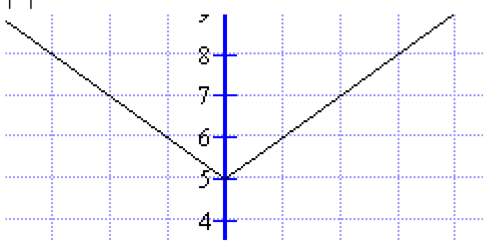
Unit 6 Section 7 Worksheet – ANSWERS

Write an equation for each translation of $y = |x|$.

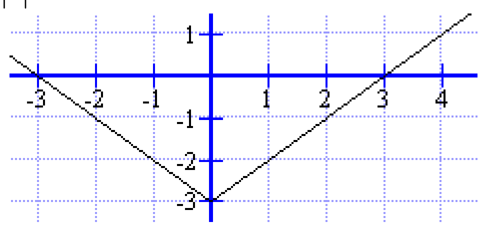
1) 7 units up $y=|x|+7$
2) 4 units down $y=|x|-4$
3) 3.5 units up $y=|x|+3.5$
4) 2 units down $y=|x|-2$
5) 3 units to the right $y=|x-3|$
6) 1 unit to the left $y=|x+1|$

Graph each function by translating $y = |x|$.

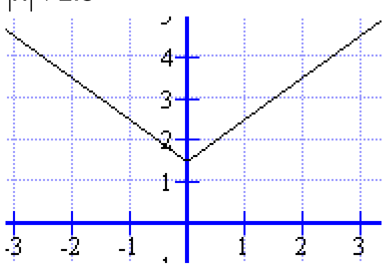
7) $y = |x| + 5$



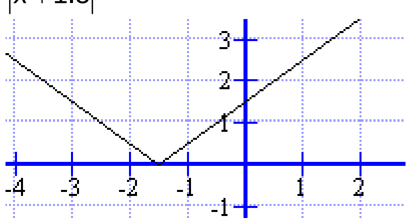
8) $y = |x| - 3$



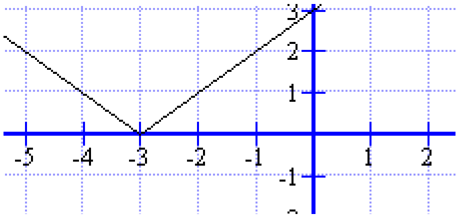
9) $y = |x| + 1.5$



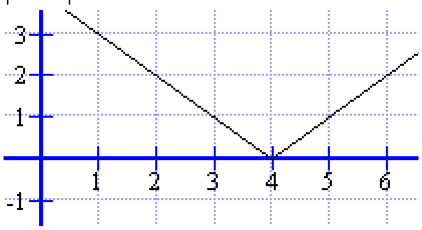
10) $y = |x + 1.5|$



11) $y = |x + 3|$



12) $y = |x - 4|$



Write an equation for each translation of $y = -|x|$.

7) 3.5 units up $\quad$ $y=-|x| + 3.5$

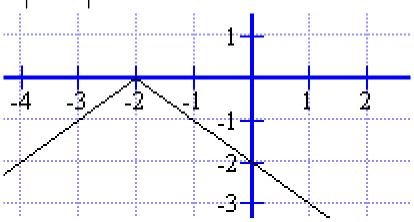
8) 5 units down $\quad$ $y=-|x| - 5$

9) 2 units to the right $\quad$ $y=-|x-2|$

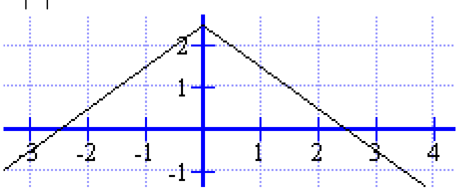
10) 3 units to the left $\quad$ $y=-|x+3|$

Graph each function by translating $y = -|x|$.

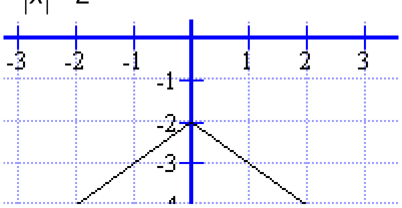
17) $y = -|x + 2|$



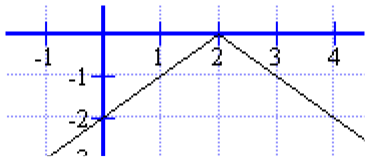
18) $y = -|x| + 2.5$



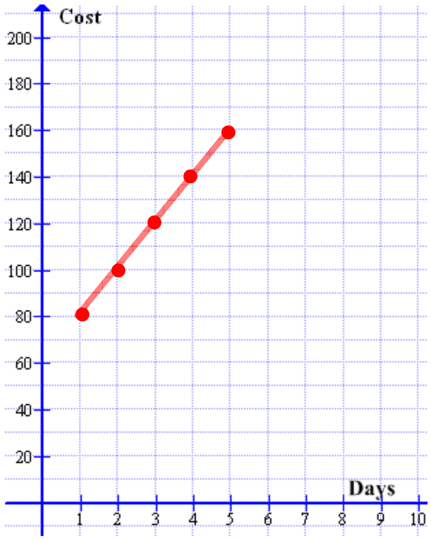
19) $y = -|x| - 2$



20) $y = -|x - 2|$


-1
1
2
3
4
-1
-2

If we graphed the ordered pairs from the table, we'd see that they all lay on a line as shown in the graph. This means they are linear.

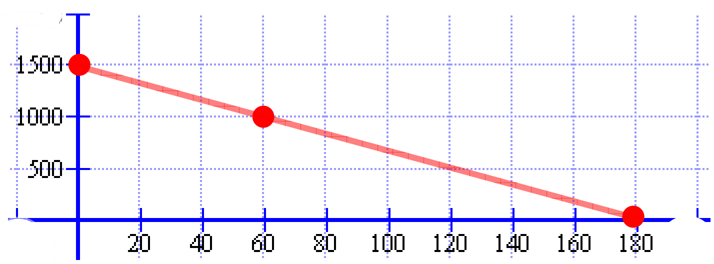


You can use the graph to find the rate of change. Recall that the independent variable is plotted along the x-axis and the dependent variable is plotted along the y-axis. We can measure the rate of change by calculating the vertical change (the rise) and dividing it by the horizontal change (the run).

$$\text{rate of change} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{change in dependent variable}}{\text{change in independent variable}}$$

EXAMPLE #2: Finding Rate of Change Using a Graph

The graph below shows the altitude of an airplane as it comes in for a landing. Find the rate of change. Explain what this rate of change means.



$$\text{rate of change} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{change in altitude}}{\text{change in time}}$$

$= \frac{1000\text{-}0}{60\text{-}180}$ Use two points on the graph

$= \frac{1000}{-120}$ Divide the vertical change by the horizontal change

$= -8\frac{1}{3}$ Simplify

The rate of change is -8 1/3 ft/sec, meaning the plane descends 8 1/3 feet each second.

OBJECTIVE #3: FINDING SLOPE USING A GRAPH

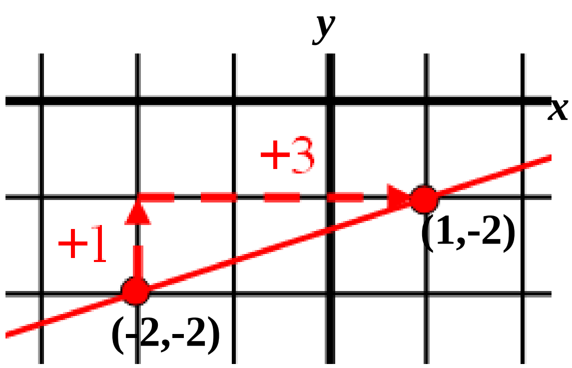
The slope of a line is its rate of change.

$$\text{Slope(m)} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in y}}{\text{change in x}} \text{ aka } \frac{\Delta y}{\Delta x}$$

EXAMPLE #3: Finding slope using a graph

Find the slope of each line

a)

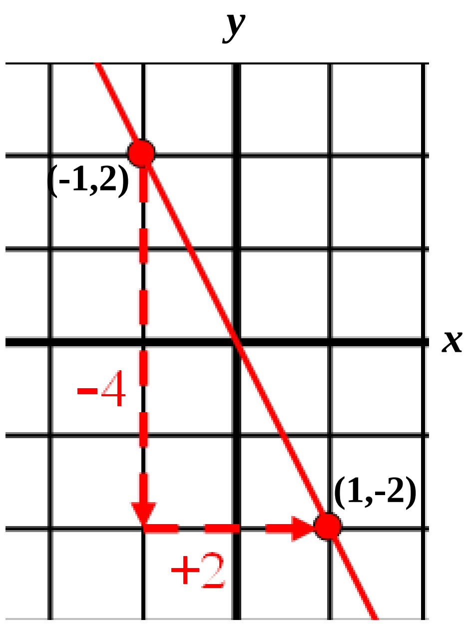


Moving from point (-2, -2) to point (1, -1) we count the y-value as rising 1 unit in a positive direction. We also count the x-value as running 3 units in a positive direction.

$$\text{slope(m)} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in y}}{\text{change in x}} = \frac{-1-(-2)}{1-(-2)} = \frac{+1}{+3}$$

The slope of the line is $\frac{1}{3}$, meaning the y-value goes up 1 for every increase of 3 in the x-value.

b)



Moving from point (-1, 2) to point (1, -2) we see the y-value going 4 in the negative direction so our vertical change is -4, while we see the x-value moving 2 in a positve direction so our horizontal change is +2.

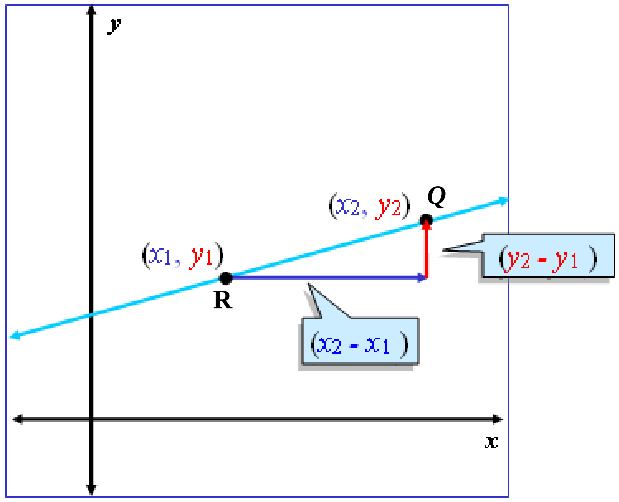
$$\text{slope(m)} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in y}}{\text{change in x}} = \frac{-2-(2)}{1-(-1)} = \frac{-4}{+2} = -2$$

The slope of this line is -2, meaning that the y-value goes down 2 for every 1 unit increase in the x-value. Notice the first value used above the dividing line comes from the same coordinate pair as the first x-value below the dividing line. It is very important to keep the starting points in order as you move from one point to the other.

Also, notice this line leans back to the left! Lines leaning that way have a negative slope. Lines leaning to the right (like the first problem we did) have a positive slope.

You can use any two points on a line to determine its slope. Subscripts are used to distinguish between two points. In the diagram below, notice (x_1,y_1) are the coordinates of R, and (x_2,y_2) are the coordinates of Q. To find the slope of $\overleftrightarrow{RQ}$, you can use the following formula.

$$\text{slope(m)} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} \text{ where } x_2 - x_1 \neq 0$$



KEYPOINT: The numerator and denominator must use the same subtraction order!

CORRECT $\Rightarrow \dfrac{y_2 - y_1}{x_2 - x_1}$ ← Subtraction order is the same!

INCORRECT $\dfrac{y_2 - y_1}{x_1 - x_2}$ ⟵ Subtraction order is different

Reminder:

$$\text{slope(m)} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} \text{ where } x_2 - x_1 \neq 0$$

EXAMPLE #4: Finding Slope Using Points

Armed with what we've just covered, you should be able to find the slope of a line when you are given the coordinate pairs for two points on the line.

Find the slope of the line thru M(-3, 2) and N(5, 6).

$$\text{slope(m)} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{slope(m)} = \frac{6 - 2}{5 - (-3)}$$

$$\text{slope(m)} = \frac{4}{8} = \frac{1}{2}$$

After simplifying, the slope of $\overleftrightarrow{MN} = \frac{1}{2}$

You can also analyze the graphs of horizontal and vertical lines. In Example 5, you'll learn why the slope of a horizontal line is 0, and the slope of a vertical line is undefined.

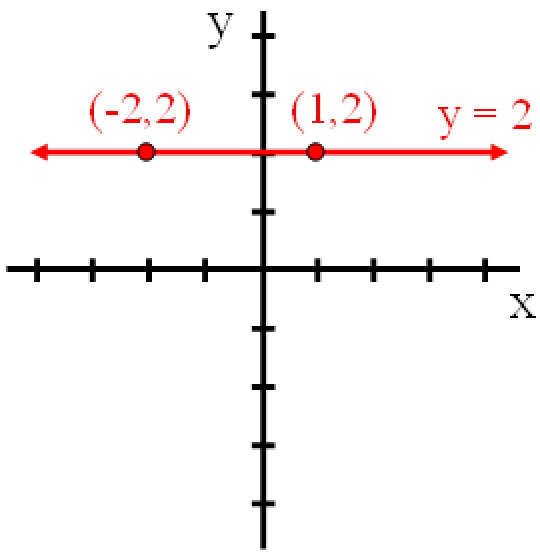
EXAMPLE #5: Horizontal and Vertical Lines

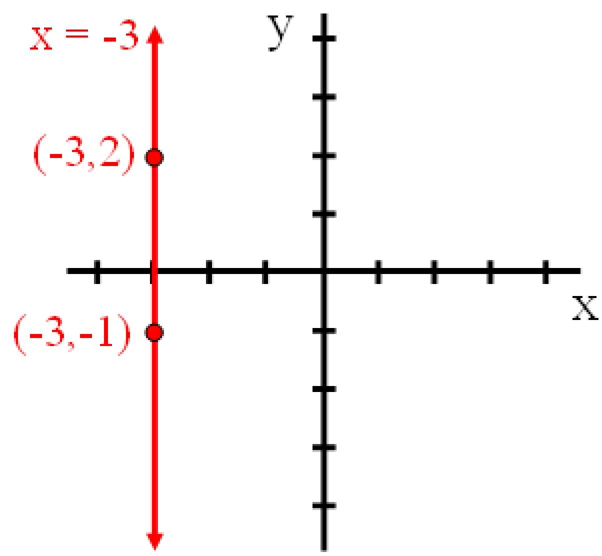
Horizontal Lines

(-2,2) (1,2)

Vertical lines

(-3,2) (-3, -1)





Now we'll look at the math!

Horizontal Lines

(-2,2) (1,2)

1 2

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{2 - 2}{1 - {}^{-}2}$$

$$m = \frac{0}{3}$$

$m = 0$

$m = 0$ for all Horizontal Lines

Vertical lines

(-3,2) (-3, -1)

1 2

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-1 - 2}{-3 - {}^{-}3}$$

$$m = \frac{-3}{0}$$

Watch Out!

m = undefined or there is "no slope"

m = undefined for all Vertical Lines

Notice that in the horizontal line, there is no change in the y-value, so the numerator is always 0 and the slope is always 0.

In the vertical line, there is no change in the x-value, so $x_2 - x_1$ always equals 0. Since we cannot divide by 0, the slope of a vertical line is always undefined!

Unit 6 Section 1 WORKSHEET

Find the rate of change for each situation.

1. A plane is traveling at 32,000 feet and begins a descent of 6,000 feet over a 45 minute time period. From there, the plane drops to 18,000 feet in 30 minutes. Compare the rates of change for both time periods.

2. It costs $96.00 for a couple to attend the amusement park. Two adults and two children can attend the amusement park for $132.00.

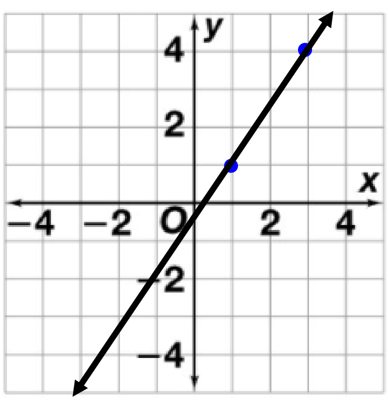
3. You drive 15 miles in 30 minutes to get to the highway and then drive 180 miles in 2.5 hours.

Find the slope of a line passing through each pair of points.

4. (2, 1), (4,7)
5. (-2, 4), (3, -6)
6. (0, 2), (-2, -8)
7. (3, 3), (-5, -5)
8. (-2, 5), (6, 5)
9. (4, 10), (4, -3)

Find the slope of each line in the graph.

10.



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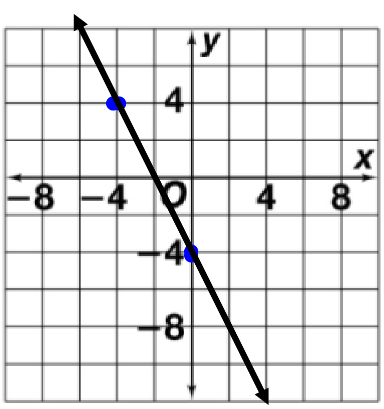

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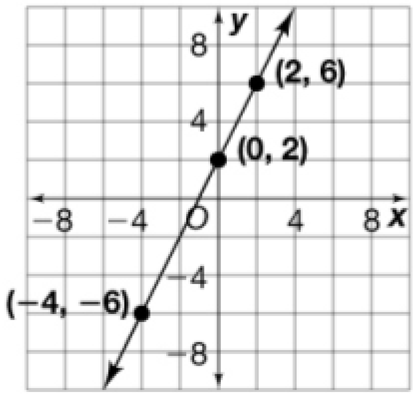

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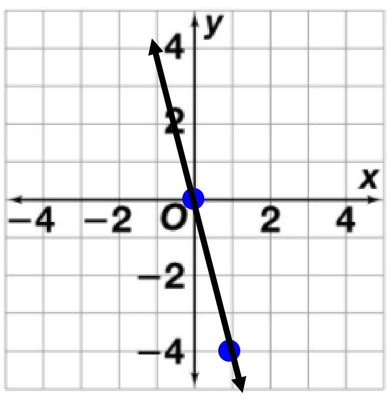

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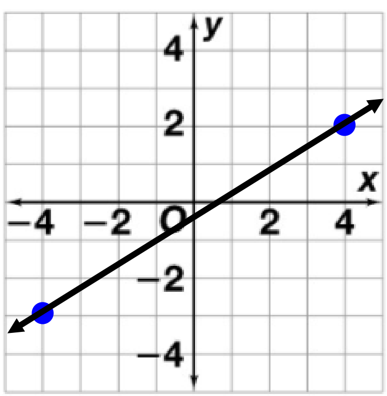


Unit 6 Section 1 WORKSHEET – ANSWER SHEET

Find the rate of change for each situation.

10. A plane is traveling at 32,000 feet and begins a descent of 6,000 feet over a 45 minute time period. From there, the plane drops to 18,000 feet in 30 minutes. Compare the rates of change for both time periods. Dropping at a rate of 8000 ft./hr then changing to a descent rate of 16,000 ft./hr

11. It costs $96.00 for a couple to attend the amusement park. Two adults and two children can attend the amusement park for $132.00. $18.00 per child

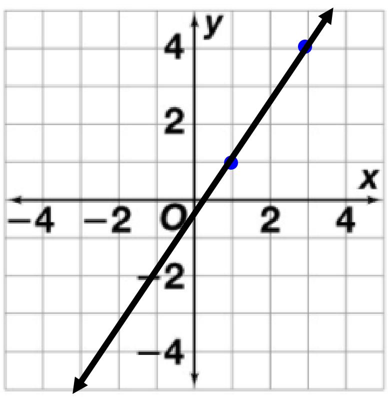
12. You drive 15 miles in 30 minutes to get to the highway and then drive 180 miles in 2.5 hours. 30 mi/hr to a rate of 72 mi/hr

Find the slope of a line passing through each pair of points.

13. (2, 1), (4,7) slope= 3
14. (-2, 4), (3, -6) slope= -2
15. (0, 2), (-2, -8) slope= 5
16. (3, 3), (-5, -5) slope= 1
17. (-2, 5), (6, 5) slope= 0
18. (4, 10), (4, -3) undefined

Find the slope of each line in the graph.

10.



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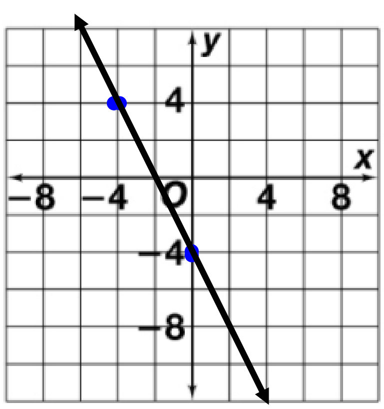
11.



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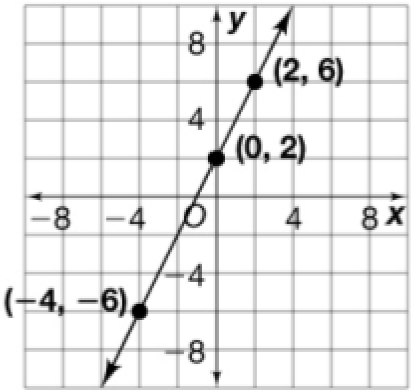

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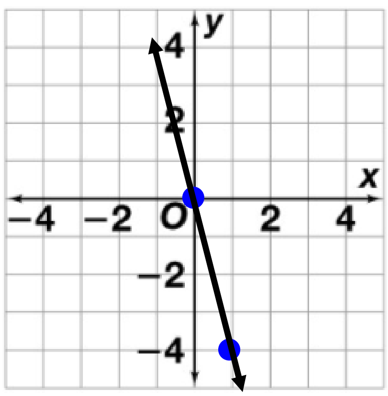

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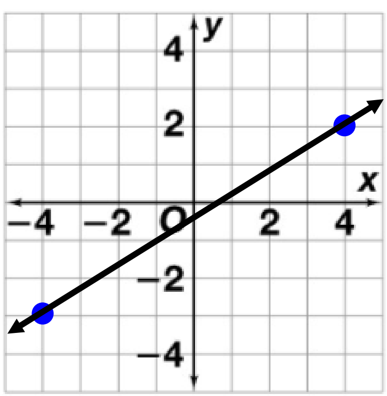

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