Unit 7—Systems of Equations and Inequalities

Section 1--Solving Systems by Graphing (pg. 402 – 405)

Vocabulary:

Systems of Linear Equations (System of Simultaneous Linear Equations) – Two or more linear equations using the same variables.

Ex) Y = 2X + 4 (Notice that both equations use the variables X and Y)
Y = 4X + 1

Solutions of the System of Linear Equations (Common Solution) – Any ordered pair in a system that makes all the equations of that system true.

Ex) (-1,5) is a solution of the system Y = 2X + 7 and Y = X + 6

Y = 2X + 7	substitute (-1,5)	Y = X + 6
5 = 2(-1) + 7	For (X,Y)	5 = -1 + 6
5 = -2 + 7		5 = 5
5 = 5		

Independent Equations (System of Consistent Equations) – When a system of linear equations has one common solution.

Ex) Y = 2X + 1
Y = 3X + 3

No Solution (System of Inconsistent Equations) – When the graphs of the equations in a system are parallel with no point of intersection.

Ex) There is no solution to the system of equations Y = 2X + 6 and 4X – 2Y = 8

(By placing both equations in slope-intercept form, it is easier to determine what the slope and y-intercepts are for each equation. If the two equations have matching slopes, but different y-intercepts, the system will not have a solution.)

Y = mX + b	Y = 2X + 6	4X – 2Y = 8
m = slope	m = 2	-4X -4X
b = y-intercept	b = 6	-2Y = -4X + 8
		-2 -2 -2
		Y = 2X – 4
		m = 2
		b = -4

Infinitely Many Solutions (System of Dependent Equations) – The number of solutions for a system of equations in which the graphs of the equations are the same line.

Ex) The system Y = 3X + 4 and -12X + 4Y = 16 has infinitely many solutions.

(By placing both equations in slope-intercept form, it is easier to determine what the slope and y-intercepts are for each equation. If the two equations have matching slopes and y-intercepts, the system have infinitely many solutions.)

Y = mX + b	Y = 3X + 4	-12X + 4Y = 16
m = slope	m = 3	+12X +12X
b = y-intercept	b = 4	4Y = 12X + 16
		4 4 4
		Y = 3X + 4
		m = 3
		b = 4

Unit 7 Section 4

Applications of Linear Systems (pg.412 – 427)

In order to solve a word problem, you must first choose a method from the first three sections. Below is a list of the sections that were covered:

Graphing (Section 1) – Look for any point common to all the lines. Any ordered pair in a system that makes all the equation true is a solution o the system of linear equations.

Substitution (Section 2) – A method of solving a system of equations by replacing one variable with an equivalent expression containing the other variable.

Elimination (Section 3) – A method for solving a system of linear equations. You add or subtract the equations to eliminate a variable.

Practice (Graphing):

1) Suppose you are testing two fertilizers on palm trees A and B, which are growing under the same conditions. Palm tree A is 8 cm tall growing at a rate of 6 cm/day. Palm tree B is 12 cm tall growing at a rate of 4 cm/day. After how many days will the palm trees be the same height?

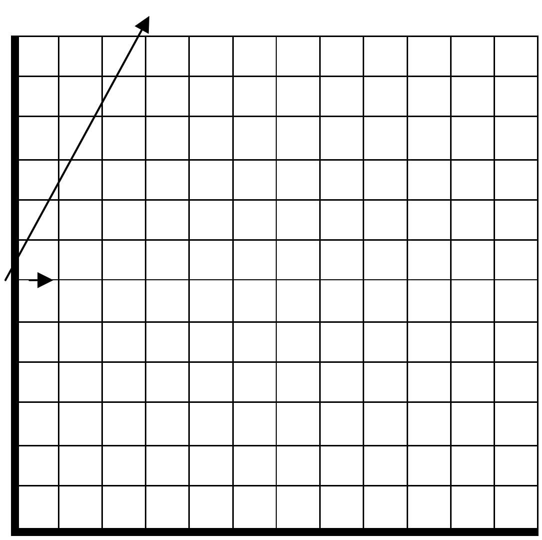
Determine Variables:
D = Days
H = Height

Write an Expression:
Palm tree height is the original height plus its daily growth

Palm Tree (A): H = 8 + 6D
Palm Tree (B): H = 12 + 4D

Write in Slope – Intercept Form:
Palm Tree (A): H = 6D + 8 Slope is 6, Y – Intercept is 8
Palm Tree (B): H = 4D + 12 Slope is 4, Y – Intercept is 12

(You might want to graph the height in two's and the days in one's)



After two days the Palm Trees will be the same height.

Practice (Substitution):

2) Your school committee is planning an after-school trip with 193 people to compete at another school. There are eight drivers available and two types of vehicles, school buses and minivans. The school bus seats 51 people each and the minivan fits 8 people each. How many buses and minivans are needed for this trip?

B = Buses
M = Minivans

Drivers: Buses and Minivans total to eight drivers

B + M = 8 — Solve for a variable, either one.
-B -B
M = -B + 8

People: People that fit on the buses and the people that fit on the vans are the total traveling

51B + 8M = 193
51B + 8(-B + 8) = 193 — Substitute the M – variable with –B + 8
51B -8B + 64 = 193 — Solve for the B - variable
43 B + 64 = 193
- 64 -64
$\frac{43\,B}{43} = \frac{129}{43}$
B = 3

There are 3 Buses that are planned for this trip.

B+ M = 8 — Use this equation to find the number of minivans needed
3 + M = 8 — Solve for the M – variable
-3 -3
M = 5

There are 5 Minivans that are planned for this trip.

Check:
Each bus holds 51 people so: 3(51) = 153 people
Each minivan holds 8 people so: 5(8) = 40 people
Total = 193 people

Practice (Elimination):

3) Suppose you bought supplies for a party. Three rolls of streamers and 15 party hats, that all cost $30. Later, you buy two rolls of streamers and four party hats, totaling $11. How much did that streamers cost? How much did the hats cost?

H = hats
S = streamers

3s + 15h = 30 — Equations for each time you bought party supplies
2s + 4h = 11

2(3s + 15h = 30) — Use multiplication to cause an additive inverse, eliminating the s
-3(2s + 4h = 11)

6s + 30h = 60 — Solve for the H - variable

<u>-6s – 12h = -33</u>

$$\frac{18h}{18} = \frac{27}{18}$$

H = 1.5

2s + 4h = 11
2s + 4(1.5) = 11
2s + 6 = 11
- 6   -6

$$\frac{2s}{2} = \frac{5}{2}$$

S = 2.5

Pick from either of the original equations and solve for the eliminated variable by replacing the H – variable with 1.5
Solve for the S – variable

Hats cost: $1.50 each
Streamers cost: $2.50 each

Check:
Hats: 1.50(19) = $28.50
Streamers: 2.50(5) = $12.50
Total = $41.00

Extra Practice:
Complete the following problems.
For numbers 1 – 2 solve by graphing.

1) A small business invests $10,000 in equipment to produce a product. Each unit of the product costs $0.65 to produce and is sold for $1.20. How many items must be sold before the business breaks even?

2) Suppose you are setting up a small business and have invested $16,000 to produce an item that will sell for $5.95. If each unit can be produced for $3.45, how many units must you sell to break even?

For numbers 3 – 4, solve by substitution.

3) A total of $12,000 is invested in two funds paying 9% and 11% simple interest. If the yearly interest is $1180, how much of the $12,000 is invested at each rate?

4) A total of $34,000 is invested in two funds paying 7% and 7.5% simple interest. If the yearly interest is $2480, how much of the $34,000 is invested at each rate?

For numbers 5 – 6, solve by elimination.

5) An airplane flying into a head wind travels the 1800 – mile flying distance between two cities in 3 hours and 36 minutes. On the return flight, the distance is traveled in 3 hours. Find the ground speed of the plane and the speed of the wind, assuming that both remain at constant.

6) Suppose you a re the manager of a shoe store. On Sunday morning you are going over the receipts for the previous week's sales. Two hundred and forty pairs of tennis shoes were sold. One style sold for $66.95 and the other sod for $84.95. The total receipts were $17,652. The cash register that was supposed to keep track of the number of each type of shoe sold malfunctioned. Can you recover the information? If so, how many of each type were sold?

Unit 7 Section 3
Solving Systems Using Elimination (pg. 405 – 409)

Vocabulary:

<u>Elimination Method</u> – A method for solving system of linear equations. You add or subtract the equations to eliminate a variable.

Ex) Solve by elimination. Check your solution.

2X – 7Y = 3
-2X + Y = -9

Step 1 – Eliminate X because the sum of the coefficients of X is zero.

2X – 7Y = 3
-2X + Y = -9 — Solve for the Y - variable
- 6Y = - 6
-6 -6
Y = 1

Step 2 – Solve for the eliminated variable X using either of the original equations.

-2X + Y = -9
-2X + 1 = -9 — Substitute the Y – variable with 1
- 1 - 1 — Solve for the X – variable
-2X = -10
-2 -2
X = 5

Since X = 5 and Y = 1, the solution is (5,1)

Step 3 – Check your solution to both equations.

2X – 7Y = 3	Substitute (5,1)	-2X + Y = -9
2(5) – 7(1) = 3	for (X,Y)	-2(5) + 1 = -9
10 – 7 = 3		-10 + 1 = -9
3 = 3		-9 = -9

Practice:

1) Solve by elimination. Check your solution.

3X + 6Y = 6
2X + Y = 1

Step 1 – Use Multiplication to create additive inverse. Eliminate Y because the sum of the coefficients of Y are zero.

3X + 6Y = 6
-6(2X + Y = 1) — Multiply the entire equation of 2X + Y = 1 by -6 to turn the Y into -6Y, allowing the elimination of Y to occur

3X + 6Y = 6
-12X – 6Y = -6 — Used the Distributive Property to second equation to eliminate parentheses
-9X = 0 — Solve for the X – variable
X = 0

Step 2 – Solve for the eliminated variable Y using either of the original equations.

2X + Y = 1
2(0) + Y = 1 — Substitute the X – variable with 0
0 + Y = 1 — Solve for the Y – variable
Y = 1

Since X = 0 and Y = 1, the solution is (0,1)

Step 3 – Check your solution to both equations.

3X + 6Y = 6	Substitute (0,1)	2X + Y = 1
3(0) + 6(1) = 6	for (X,Y)	2(0) + 1 = 1
0 + 6 = 6		0 + 1 = 1
6 = 6		1 = 1

2) Solve by elimination. Check your solution.

$$X - 8Y = -2$$

$$\frac{3X}{2Y} + 1 = \frac{10}{Y}$$

Step 1 – Use multiplication to create additive inverse. Eliminate the fraction first because the coefficients of the two fractions are zero.

$$\frac{-3}{2Y}\left(\frac{X}{1} - \frac{8Y}{1} = \frac{-1}{1}\right)$$ Multiply first equation with $\frac{-3}{2Y}$

$$\frac{-3X}{2Y} + \frac{24Y}{2Y} = \frac{6}{2Y}$$

$$\frac{-3X}{2Y} + 12 = 3Y$$ First equation re-written to eliminate fraction

$$\frac{3X}{2Y} + 1 = \frac{10}{Y}$$ Divide out the 10 and Y

$$\frac{3X}{2Y} + 1 = 10Y$$ Second equation re-written to eliminate fraction

$$\frac{-3X}{2Y} + 12 = 3Y$$ Both equations re-written and combined to eliminate The fraction

$$\frac{3X}{2Y} + 1 = 10Y$$

$$\frac{13}{13} = \frac{13Y}{13}$$ Solve for the Y – variable

$$1 = Y$$

Step 2 – Solve for the eliminated variable X using either of the original equations.

X – 8Y = -2	
X – 8(1) = -2	Substitute the Y – variable with 1
X – 8 = -2	Solve for the X – variable
+8 +8	
X = 6	

Since X = 6 and Y = 1, the solution is (6,1)

Step 3 – Check your solution to both equations.

X – 8Y = -2 Substitute (6,1) $\frac{3X}{\ } \quad \frac{10}{\ }$

6 – 8(1) = -2 for (X,Y)
6 – 8 = -2
-2 = -2

2Y + 1 = Y
3(6) 10
2(1) + 1 = 1

18
2 + 1 = 10
9 + 1 = 10
10 = 10

Extra Practice:

Complete the following problems.

For questions 1 – 10, solve each system by elimination. Check your solution.

1) 5X – 2Y = 7
X – 3Y = -9

2) 3X – 4Y = 5
2Y + X = 3

3) 5X + 9Y = 112
3X – 2Y = 8

4) 8X – 2Y = 58
6X – 2Y = 40

5) X + 5Y = -7
2X + 7Y = -8

6) Y = 5X + 1
2Y = -5X + 2

7) 3X + 2Y = -9
-10X + 5Y = -5

8) 9X + 5Y = 34
8X – 2Y = -2

9) X = 12Y – 14
3Y + 2X = 26

10) 8X + 3Y = -21
4X + 5Y = -7

Unit 7 Section 2
Solving Systems Using Substitution (pg. 410 – 412)

Vocabulary:

Substitution Method – A method of solving a system of equations by replacing one variable with an equivalent expression containing the other variable. (You must have one equation that has a variable alone. If this is not the case, you must choose an equation and solve for a variable first).

Ex) Solve using substitution. Check your answer.

X + Y = 5
X = Y + 7

Step 1 – Write an equation containing only one variable, solve it. (Pick one equation, does not matter which one).

X + Y = 5	
(Y + 7) + Y = 5	Substitute the X – variable with Y + 7
2Y + 7 = 5	Combine like terms
-7 -7	Solve for the Y – variable
$\frac{2Y}{2} = \frac{-2}{2}$	
Y = -1	

Step 2 – Solve for the other variable. (Pick one equation, does not matter which one).

X = Y + 7	
X = -1 + 7	Substitute the Y – variable with -1
X = 6	Solve for the X – variable

Since X = 6 and Y = -1, the solution is (6,-1)

Step 3 – Check your solution to both equations.

X + Y = 5	Substitute (6,-1)	X = Y + 7
6 + -1 = 5	for (X,Y)	6 = -1 + 7
5 = 5		6 = 6

Practice:

1) Solve using substitution. Check your solution.

3X – Y = 7
Y = X + 3

Step 1 – Write an equation containing only one variable, solve it. (Pick one equation, does not matter which one).

3X – Y = 7	
3X – (X + 3) = 7	Substitute the Y – variable with X + 3
3X – X – 3 = 7	Use the Distributive Property to eliminate the parentheses
2X – 3 = 7	Combine like terms
+ 3 + 3	Solve for the X - variable
$\frac{2X}{2} = \frac{10}{2}$	
X = 5	

Step 2 – Solve for the other variable. (Pick one equation, does not matter which one).

Y = X + 3	
Y = 5 + 3	Substitute the X – variable with 5

Y = 8 Solve for the Y – variable

Since X = 5 and Y = 8, the solution is (5,8)

Step 3 – Check your solution to both equation.

3X – Y = 7	Substitute (5,8)	Y = X + 3
3(5) – 8 = 7	for (X,Y)	8 = 5 + 3
15 – 8 = 7		8 = 8
7 = 7		

2) Solve using substitution. Check your solution.
3X + 4Y = 26
-2X + Y = 1

Step 1 – Write an equation containing only one variable, solve it. (Pick one equation, does not matter which one).

-2X + Y = 1 Re-Write the equation with one variable alone
+ 2X + 2X
Y = 2X + 1

3X + 4Y = 26 Two equations to now use
Y = 2X + 1

3X + 4Y = 26	
3X + 4(2X + 1) = 26	Substitute the Y – variable with 2X + 1
3X + 8X + 4 = 26	Use Distributive Property to eliminate the parentheses
11X + 4 = 26	Combine like terms
- 4 - 4	Solve for the X – variable
$\frac{11X}{11} = \frac{22}{11}$	
X = 2	

Step 2 – Solve for the other variable (Pick one equation, does not matter which one)

-2X + Y = 1	
-2(2) + Y = 1	Substitute the X – variable with 2
-4 + Y = 1	Solve for the Y – variable
+ 4 + 4	
Y = 5	

Since X = 2 and Y = 5, the solution is (2,5)

Step 3 – Check your solution to both equations.

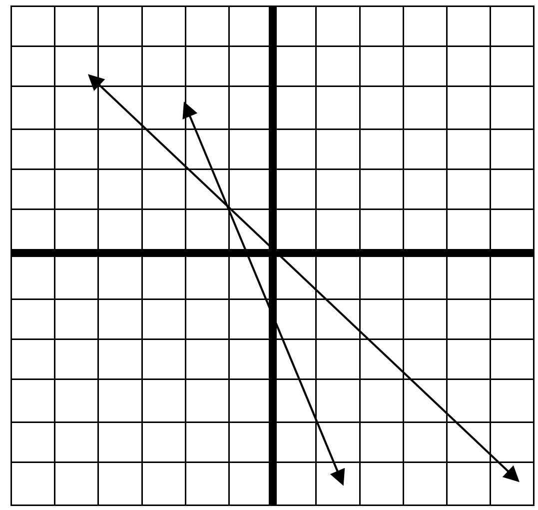
3X + 4Y = 26	Substitute (2,5)	-2X + Y = 1
3(2) + 4(5) = 26	for (X,Y)	-2(2) + 5 = 1
6 + 20 = 26		-4 + 5 = 1
26 = 26		1 = 1

3) Graph the system to estimate the solution. Check your answer to see if the point of intersection is the solution to the system. Write no solution or infinitely many solutions if appropriate.

X + Y = 0
5X + 2Y = -3

Step 1 – Put equations in Slope – Intercept From (Y = mX + b), graph.

X + Y = 0	5X + 2Y = -3
-X -X	- 5X - 5X
Y = -X + 0	$\frac{2Y}{2} = \frac{-5X}{2} - \frac{3}{2}$
Y = -X + 0	$Y = \frac{-5}{2}X - 1.5$
M = -1	$M = \frac{-5}{2}$
B = 0	B = -1.5



Point of intersection is (-1,1)

Step 2 – Check your solution to both equation.

X + Y = 0	Substitute (-1,1)	5X + 2Y = -3
-1 + 1 = 0	for (X,Y)	5(-1) + 2(1) = -3
0 = 0		-5 + 2 = -3
		-3 = -3

Extra Practice:
Complete the following problems.
For question 1 – 6, solve each system using substitution. Check your solution.

1) 4X – 7Y = 9
   Y = X + 3

2) 3X – 4Y = -5
   X = Y + 2

3) 5X - 2Y = 7
   X – 3Y = -9

4) X – 7Y = 0
   3X – 1 = 14Y

5) 2X + 7Y = -1
   3X + Y = 8

6) 3X – 3 = 5Y
   2X + Y = 15

For questions 7 – 12, graph each system to estimate the solution. Check your solution to both equations. Write no solution or infinitely many solutions where appropriate.

7) Y = -X + 4
   Y = 2X + 6

8) 4X + 2Y = 8
   Y = -2X + 4

9) Y = X – 5
   2X + 3Y = 0

10) X – Y = 1
    2X + Y = 8

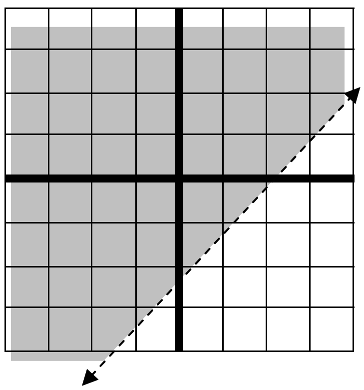
11) $Y = -\frac{2}{3}X + 4$
    2X + 3Y = -6

12) Y = 3X – 2
    Y = -2X + 4

Unit 7 Section 5 – Linear Inequalities and
Systems of Linear Inequalities (pg. 427 – 430)

Vocabulary:

Linear Inequality – A mathematical sentence that describes a region of the coordinate plane having a boundary line. Each point in the region is a solution of the inequality.

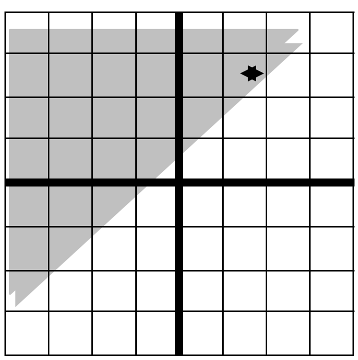


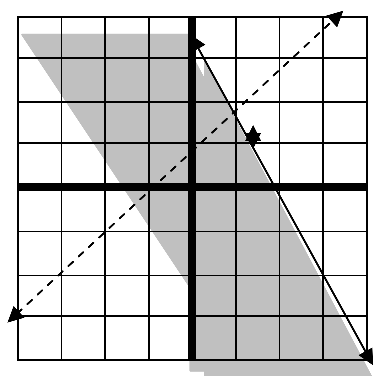
Ex) Y > X - 2

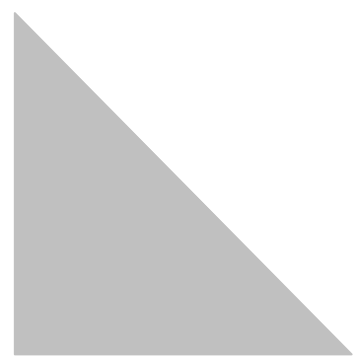
Solutions of Inequality – Any value or values of a variable in the inequality that makes an inequality true. Use any point to test the original inequality for shading.

Ex) The solution of the inequality X ≥ -2, is -2 and all numbers greater than -2.

Ex)







Y > X + 1
Each point on a dashed boundary Is not a solution.

Y ≤ -2X + 4
Each point on a solid boundary line is a solution.

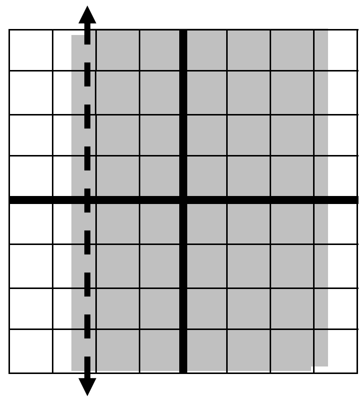
A common point to test the inequality for shading purposes is the point (0,0). The graph on the left ends up being false so you must shade away from the point (0,0). The graph on the right ends up being true, so you must shade towards the point (0,0).

Practice:

Sketch the graphs of the following linear inequalities.

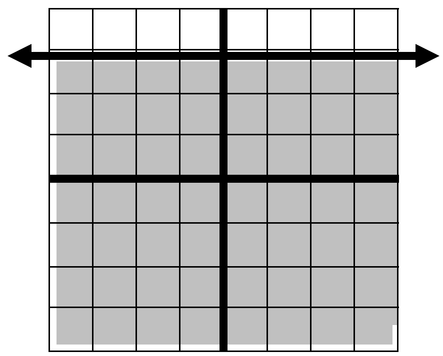
1) X > -2

Testing Point (0,0)
X > -2
0 > -2
True



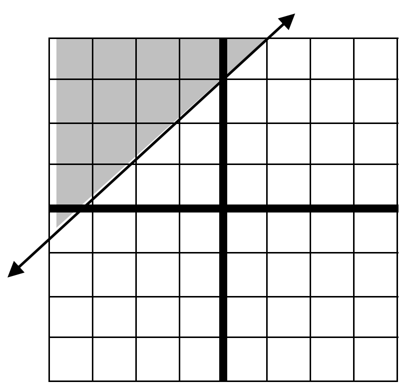
2) Y ≤ 3

Testing Point (0,0)
Y ≤ 3
0 ≤ 3
True



3) Y ≥ X + 3
M = 1
B = 3

Testing Point (0,0)
Y ≥ X + 3
0 ≥ 0 + 3
0 ≥ 3
False



4) $3X + Y < 2$

$\underline{-3X \qquad -3X}$

$Y < -3X + 2$

$M = -3$

$B = 2$

Testing Point (0,0)

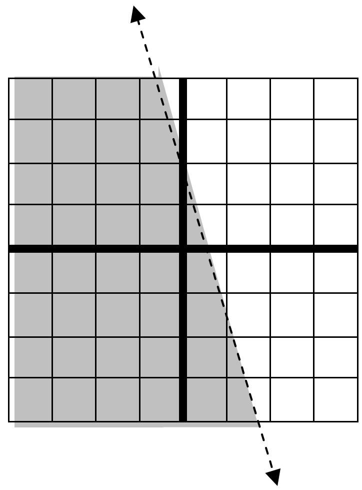
$3X + Y < 2$

$3(0) + 0 < 2$

$0 + 0 < 2$

$0 < 2$

True



5) $5X – 2Y \leq 10$

$\underline{-5X \qquad -5X}$

$\frac{-2Y}{-2} \leq \frac{-5X}{-2} + \frac{10}{-2}$

$Y \geq \frac{5}{2} X – 5$

$M = \frac{5}{2}$

$B = -5$

Testing Point (0,0)

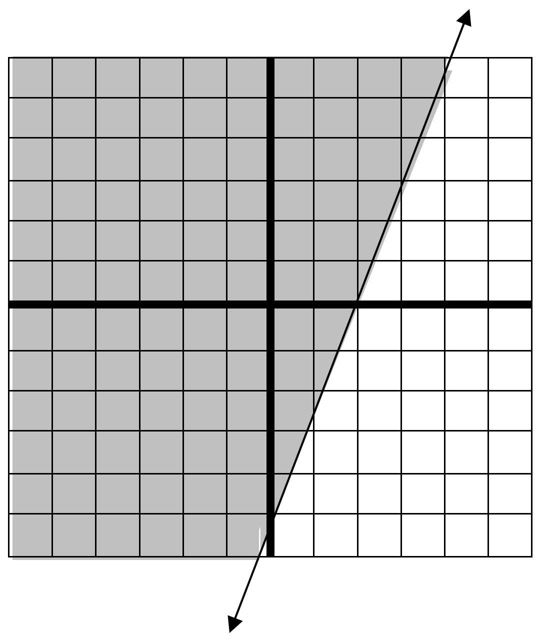
$5X – 2Y \leq 10$

$5(0) – 2(0) \leq 10$

$0 – 0 \leq 10$

$0 \leq 10$

True



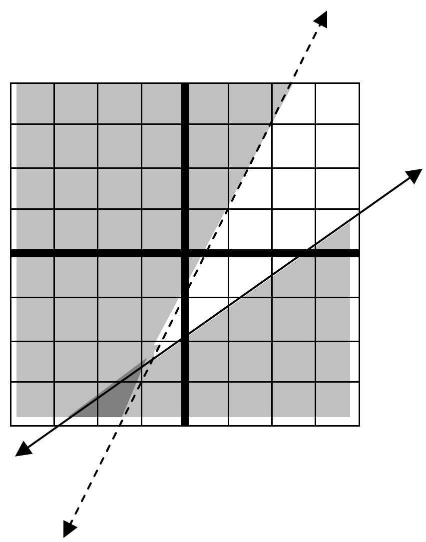
Vocabulary:

<u>Systems of Linear Inequalities</u> – Two or more linear inequalities using the same variables.

Ex) $Y < 2X - 2$
$Y \geq -X$

<u>Solution of a System of Linear Inequalities</u> – Any ordered pair that makes all of the inequalities in the system true.

Ex) Solve by graphing:
$Y > 2X - 1$
$2X - 3Y \geq 6$



$Y > 2X - 1$
$M = 2$
$B = -1$

$$2X - 3Y \geq 6$$
$$\underline{-2X \qquad -2X}$$
$$\frac{-3Y}{-3} \geq \frac{-2X}{-3} + \frac{6}{-3}$$
$$Y \leq \frac{2}{3}X - 2$$
$$M = \frac{2}{3}$$
$$B = -2$$

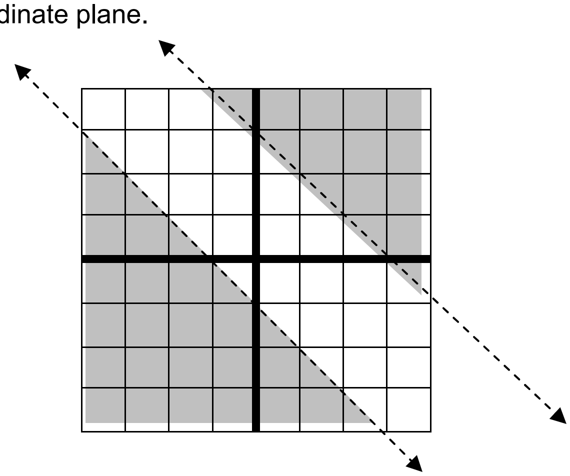
Testing Point (0,0)
$Y > 2X - 1$
$0 > 2(0) - 1$
$0 > 0 - 1$
$0 > -1$
True

Testing Point (0,0)
$2X - 3Y \geq 6$
$2(0) - 3(0) \geq 6$
$0 - 0 \geq 6$
$0 \geq 6$
False

Practice:

Graph the solution set of each system in a coordinate plane.

1) $X + Y > 3$
$X + Y < -1$



$X + Y > 3$
$\underline{-X \qquad -X}$
$Y > -X + 3$
$M = -1$
$B = 3$

$X + Y < -1$
$\underline{-X \qquad -X}$
$Y < -X - 1$
$M = -1$
$B = -1$

Testing Point (0,0)
$X + Y > 3$
$0 + 0 > 3$
$0 > 3$
False

Testing Point (0,0)
$X + Y < -1$
$0 + 0 < -1$
$0 < -1$
False

2) $X + Y \le 1$
$-X + Y \le 1$
$Y \ge 0$

$X + Y \le 1$
$-X \quad -X$
$Y \le -X + 1$
M = -1
B = 1

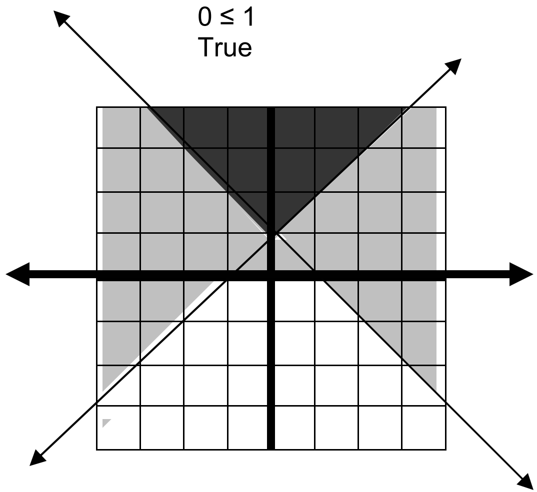
Testing Point (0,0)
$X + Y \le 1$
$0 + 0 \le 1$
$0 \le 1$
True

$-X + Y \le 1$
$+X \quad +X$
$Y \le X + 1$
M = 1
B = 1

Testing Point (0,0)
$-X + Y \le 1$
$0 + 0 \le 1$
$0 \le 1$
True

$Y \ge 0$

Testing Point (0,0)
$Y \ge 0$
$0 \ge 0$
True



Extra Practice:

Complete the following problems.

Graph the solution set of each system in a coordinate plane.

1) $2X + Y < 2$
$6X + 3Y > 2$

2-6 (See graphs in answer keys; have students write equations.)

Practice:

1) **Is (2,5) a solution of the following system Y = 2X + 1 and 2X – Y = 8?**

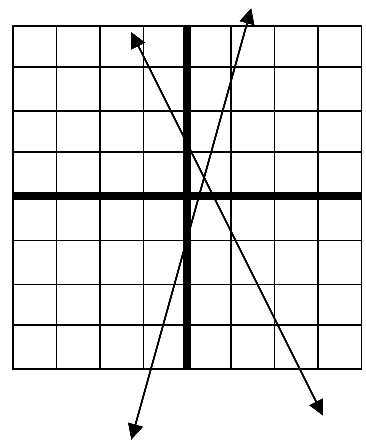
Y = 2X + 1	substitute (2,5)	2X – Y = 8
5 = 2(2) + 1	for (X,Y)	2(2) – 5 = 8
5 = 4 + 1		4 – 5 = 8
5 = 5		-1 ≠ 8

Not a Solution

2) **Solve by graphing. Write no solution or infinitely many solutions where appropriate.**

4X – 1 = Y
Y = 4X – 1
m = 4
b = -1

2X + Y = 2
-2X -2X
Y = -2X + 2
m = -2
b = 2



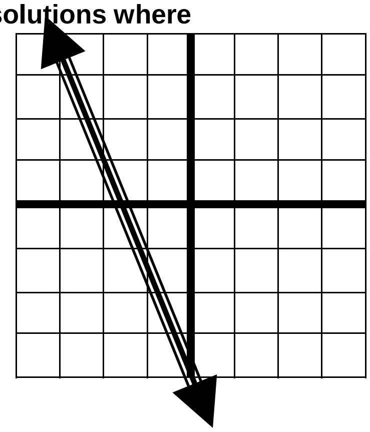
Check: Make sure (.5,1) makes both equations true.

4X – 1 = Y	substitute (.5,1)	2X + Y = 2
4(.5) – 1 = 1	for (X,Y)	2(.5) + 1 = 2
2 – 1 = 1		1 + 1 = 2
1 = 1		2 = 2

3) **Solve by graphing. Write no solution or infinitely many solutions where appropriate.**

Y = -3X – 4
m = -3
b = -4

3X + Y = -4
-3X -3X
Y = -3X – 4
m = -3
b = -4



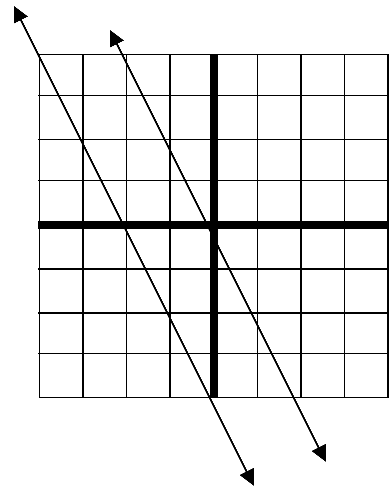
Infinitely Many Solutions

4) **Solve by graphing. Write no solution of infinitely many solutions where appropriate.**

2X + Y = 0
-2X -2X
Y = -2X + 0
m = -2
b = 0

Y = -2X – 4
m = -2
b = -4

No Solution



Extra Practice:
Complete the following problems.

For questions 1-4, tell if (1,2) is a solution of each system.

1) 4X + 2Y = 8 and Y = -2X – 3
2) Y = 3X – 1 and –X + Y = 1
3) 4X – 2Y = 0 and Y = -5X + 7
4) Y = 2X – 7 and -7 + Y = 2X

For questions 5-8, solve by graphing.

5) X + Y = 3
X – Y = 5

6) X – Y = -4
3X – Y = -10

7) Y = 3X + 5
X + Y = 1

8) Y = X + 3
2Y = 3X + 1

For questions 9-12, decide if each system has one solution (consistent), no solution (Inconsistent) or infinitely many solutions (dependent) without graphing.

9) X + 2Y = 6
Y = -1/2X – 3

10) X – Y = -2
2X – 2Y = -4

11) 3X – 4Y = 12
5X + 4Y = -12

12) X – Y = -3/2
-2X + 5Y = -4.5