Unit 8-- Exponents and Exponential Functions

Section 1 – Zero and Negative Exponents

Properties:

Zero as an Exponent – For every nonzero number *a*, $a^0 = 1$

Take a piece of paper, does not matter what type of paper it is, and try to fold it as many times as you can (usually the highest number of folds is about seven times). While folding your paper, follow the chart below. The left hand column is the number of folds you did to your paper. The right hand column is for the number of layers you paper has because of your number of folds.

# of Folds	# of Layers
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

So when you made no folds at all, you had only one layer in front of you. When you made one fold, you then had two layers in front of you. Each time you did a fold the number of layers doubled. When the number of folds was small, so was the number of layers. It was not that hard to count them out. When we increased the number of folds, the layers increased drastically and may have been harder to count. So we are going to create a short – cut. We will do this by using exponents.

Since the number of layers doubled each time a fold was done, we will use the number *2* as our base number. They number of layers depend on the number of folds, so we will use the number of folds as our exponents. So now look at the chart below to see the break down using exponents. This chart helps justify why a zero exponent is equal to *1*.

# of Folds	# of Layers	Exponential Form	Factors
0	1	2^0	1
1	2	2^1	2
2	4	2^2	2 • 2
3	8	2^3	2 • 2 • 2
4	16	2^4	2 • 2 • 2 • 2
5	32	2^5	2 • 2 • 2 • 2 • 2
6	64	2^6	2 • 2 • 2 • 2 • 2 • 2
7	128	2^7	2 • 2 • 2 • 2 • 2 • 2 • 2

Negative Exponent – For every nonzero number *a* and integer *n*, $a^{-n} = \frac{1}{a^n}$

Look at the chart below. The numbers in the left hand column represent an exponential expression, while the numbers in the right hand column represent its value.

Unit 8 Section 7
Exponential Functions

Vocabulary:

Exponential Function – A function that repeatedly multiplies an initial amount by the same positive number. You can model all exponential functions by using y = ab^x, where a is a non-zero constant, b > 0, b ≠ 1.

Ex) $y = 3.2\,(2.4)^x$

Practice (Evaluating a function rule):

Ex) y = 3^x, for x = 4

Y = 3^4

Y = 81

Ex) f(x) = 3^x - 4, for x = 2

F(2) = 3^2 - 4

F(2) = 9 – 4

F(2) = 5

Ex) g(x) = 5^{-x} - 3, for x = -1

G(-1) = $5^{-(-1)}$ - 3

G(-1) = 5^1 - 3

G(-1) = 5 – 3

G(-1) = 2

Ex) y = 2^{-x}, for x = 4

Y = 2^{-4}

Y = $\frac{1}{2^4}$

Y = $\frac{1}{16}$

Ex) f(x) = $\left(\frac{1}{2}\right)^x$ + 2, for x = -3

F(x) = $\left(\frac{1}{2}\right)^{-3}$ + 2

F(x) = $\frac{2^3}{1}$ + 2

F(x) = 8 + 2

F(x) = 10

Practice (Making a table to evaluate domains of functions):

Ex) f(x) = 2^{-x} for x = -3, -2, -1, 0, 1, 2

F(-3) = 2^{--3} = 2^{3} = 8
F(-2) = 2^{--2} = 2^{2} = 4
F(-1) = 2^{--1} = 2^{1} = 2
F(0) = 2^{0} = 1
F(1) = 2^{-1} = $\frac{1}{2^{1}}$ = $\frac{1}{2}$

F(2) = 2^{-2} = $\frac{1}{2^{2}}$ = $\frac{1}{4}$

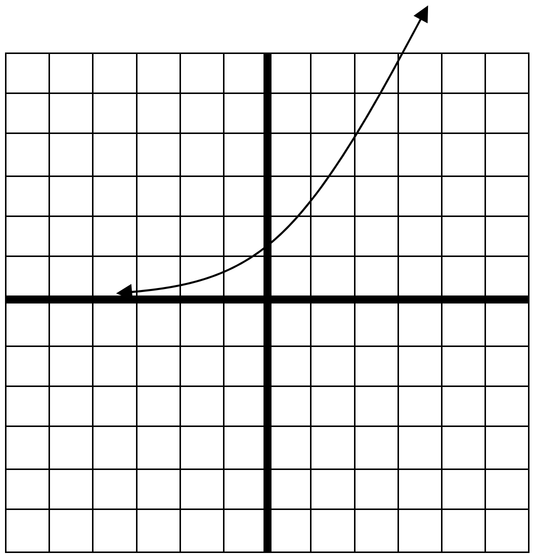
X	-3	-2	-1	0	1	2
2^{-x}	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$

Practice (Graphing exponential functions):

Ex) f(x) = 2^{x} Pick at least 5 x – values to create a table (like above)
For this example we will use -2, -1, 0, 1, 2

F(-2) = 2^{-2} = $\frac{1}{2^{2}}$ = $\frac{1}{4}$
F(-1) = 2^{-1} = $\frac{1}{2^{1}}$ = $\frac{1}{2}$
F(0) = 2^{0} = 1
F(1) = 2^{1} = 2
F(2) = 2^{2} = 4

X	-2	-1	0	1	2
F(x) = 2^{x}	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4



Extra Practice

For questions 1 – 6, evaluate the function rule.

1) g(x) = 7^x + 7, for x = 2
2) y = 15^{x-3}, for x = 1
3) f(x) = $(-4)^x$, for x = 0
4) g(x) = 2^x - 5, for x = -4
5) f(x) = $\left(\frac{1}{3}\right)^x$ + 10, for x = 3
6) h(x) = $\left(\frac{1}{5}\right)^x$, for x = -2

For questions 7 & 8, make a table to evaluate the domains of the function.

7)

X	-2	-1	0	1	2	3
G(x) = 4^x						

8)

X	-3	-2	-1	0	1	2
F(x) = 4^{-x}						

For questions 9 & 10, graph the exponential functions.

9) g(x) = $2^x + 3$

10) h(x) = $\left(\frac{1}{2}\right)^x - 2$

Unit 8 Section 6
Geometric Sequences

Vocabulary:

Geometric Sequence – A number sequence formed by multiplying a term in a sequence by a fixed number to find the next term.

Ex) 9, 3, 1, $\frac{1}{3}$, ... This is a geometric sequence because you divide the previous number by 3 to get to the next number.

Common Ratio – The fixed number used to find terms in a geometric sequence.

Ex) In the example above, the common ratio is $\frac{1}{3}$, because you are dividing each number by three (or taking $\frac{1}{3}$ of the previous number)

Arithmetic Sequence – A number sequence formed by adding a fixed number to each previous term.

Ex) 4, 7, 10, 13, ... This is an arithmetic sequence because you add the previous number by 3 to get to the next number.

Practice (Finding a common ratio):

Ex) 10, 100, 1000, 10000, ... The common ratio is 10

Practice (Finding the next three terms in a sequence):

Ex) 2, -4, 8, -16, ... The common ratio is –2
Next 3 terms: 32, -64, 128

Practice (Determine if it's an arithmetic or geometric sequence):

Ex) 7, 21, 63, 189, . . . Geometric because you multiply by 3 to get to the next number.

Ex) 10, 5, 0, -5, -10, . . . Arithmetic because you add –5 to get to the next number.

Practice (Finding terms of a sequence):

$A(n) = a \cdot r^{n-1}$
A(n) = nth term
A = first term
R = common ratio
N(exponent) = term number

Ex) Find the first, fifth and tenth terms of the sequence that has the rule:
$A(n) = 5(-2)^{n-1}$
First term: $A(1) = 5(-2)^{1-1} = 5(-2)^{0} = 5 \cdot 1 = 5$
Fifth term: A(5) = 5(-2)´ $^{-1}$ = 5 @2^{a4} = 5 • 16 = 80
Tenth term: A(10) = $5(-2)^{10-1}$ = 5 @2^{a9} = 5 • -512 = -2560

Extra Practice:

For questions 1 – 3, find the common ratio.

1) -6, -12, -24, -48, . . .
2) 10, 5, 2.5, 1.25, 0.625, . .
3) -80, 20, -5, 1.25, . .

For questions 4 – 6, find the next three terms of the sequence.

4) -2, 10, -50, 250, . .
5) $\frac{1}{2}, \frac{3}{4}, \frac{9}{8}, \frac{27}{16}, . .$
6) 1, 3, 9, 27, . .

For questions 7 – 9, determine if it is an arithmetic or geometric sequence.

7) 3, 6, 9, 12, . .
8) 12, 6, 3, $\frac{3}{2}$, . .
9) -0.2, -0.02, -0.002, -0.0002, . .

For questions 10 – 12, find the fifth and eighth terms of each sequence.

10) $A(n) = -3(-2)^{n-1}$
11) $A(n) = 0.5 \cdot 4^{n-1}$
12) $A(n) = 2(-3)^{n-1}$

Unit 8 Section 5

Division Properties of Exponents (pg.155 – 158)

When dividing by the same base, keep the base and subtract the exponents.

Practice (Dividing with like bases):

Ex) $\frac{b^7}{b^4} = b^3$

Ex) $\frac{49x^9}{7x^7} = 7x^2$

Ex) $\frac{c^3}{c^2} = c^1 = c$

Ex) $\frac{14d^6}{2d} = \frac{14d^6}{2d^1} = 7d^5$

Ex) $\frac{24y^9}{12y^0} = 2y^9$ Remember that $y^0 = 2 \text{ and } 12 \cdot 1 = 12$

Ex) $\frac{x^2}{x^3} = x^{-1} = \frac{1}{x^1} = \frac{1}{x}$

Ex) $\frac{x^{-2}y^3}{x^3y^{-4}} = x^{-5}y7 = \frac{1}{x^5} \cdot \frac{y^7}{1} = \frac{y^7}{x^5}$

Ex) $\frac{9^8}{9^6 \cdot 9^2} = \frac{9^8}{9^8} = 9^0 = 1$

Extra Practice:

For questions 1 – 10, simplify each equation.

1) $\frac{y^7}{y^6}$
2) $\frac{15x^4}{5x^3}$
3) $\frac{36t^3}{3t^0}$
4) $\frac{k^2}{k^3}$
5) $\frac{r^{-4}z^5}{r^4z^{-2}}$

6) $\dfrac{x^{4} y^{-8} z^{-4}}{x^{-2} y^{3} z^{-1}}$

7) $\dfrac{m^{0} n^{3}}{m^{5} n^{-6}}$

8) $\dfrac{q^{-4} r^{0}}{q^{6} r^{-7}}$

9) $\dfrac{x^{4} y^{-2}}{x^{-3} y^{5}}$

10) $\dfrac{m^{6} n^{0} p^{-4}}{m^{-4} n^{9} p^{4}}$?

Unit 8 Section 4

More Multiplication Properties of Exponents (pg. 152 – 155)

When raising a power to a power, multiply the exponents together.

Practice:

Ex) $(c^2)^3 = c^6$

Ex) $(x^4)^{-3}(x^2)^6 = x^{-12}x^{12} = x^0 = 1$

Ex) $(r^2)^9(r^6)^0 = r^{18}\cdot 1 = r^{18}$

Ex) $(y^4)^{-5} = y^{-20} = \frac{1}{y^{20}}$

Ex) ($(z^3)^{-1}(z^2)^5 = z^{-3}z^{10} = z^7$

Ex) $(m^4n^{-3})^3(m^2n^{-2})^{-1} = (m^{12}n^{-9})(m^{-2}n^2) = m^{10}n^{-7} = \frac{m^{10}}{1}\cdot\frac{n^{-7}}{1} = \frac{m^{10}}{1}\cdot\frac{1}{n^7} = \frac{m^{10}}{n^7}$

Extra Practice:

For questions 1 – 8, simplify the expression.

1) $r^{-4}\cdot(r^4q^3)^2$
2) $(2z^0)^2(5z^3)^{-4}$
3) $7(3x^4)^2$
4) $(3y^6)^{-5}$
5) $(x^{-2})^3x^{-12}$
6) $(-2r^{-4})^2(-3r^2t8)^{-1}$
7) $(2x^2y^{-2})^5(4xy^{-3})^{-8}$
8) $2m^{-9}n^2(2m^0n^{-4})^{-6}$

Unit 8 Section 3
Multiplication Properties of Exponents (pg.152 – 155)

Multiplying Powers with the Same Base:
Keep the base, add the exponents together.

Practice (with like bases):

Ex) $5^2 \cdot 5^3 \cdot 5 = (5 \cdot 5) \cdot (5 \cdot 5 \cdot 5) \cdot (5) = 5^6$

Ex) $a^5 \cdot a^4 = (a \cdot a \cdot a \cdot a \cdot a) \cdot (a \cdot a \cdot a \cdot a) = a^9$

Ex) $r^2 \cdot r^0 \cdot r = (r \cdot r) \cdot (1) \cdot (r) = r^3$

Multiplying Powers with Different Bases:
Match up alike bases, bring all the rest down in your answer.

Practice (with different bases):

Ex) $a^2 b \cdot ab^3 = a^3 b^4$

Ex) $(-10x^6)(2x^4) = -20x^{10}$

Ex) $c^{-2} c^7 = c^5$

Ex) $5x^2 \cdot 2x^{-5} = 10x^{-7} = \frac{10}{1} \cdot \frac{x^{-7}}{1} = \frac{10}{1} \cdot \frac{1}{x^7} = \frac{10}{x^7}$

Extra Practice:

For problems 1 – 8, simplify the expression.

1) $x^0 x^4 x^6 x$
2) $(-5)(-2x^9)$
3) $(a^2 b^3)(a^5 b^4)$
4) $(4d^{-4})(-2d^7)$
5) $y^{-8} \cdot y^{-3}$
6) $3^3 \cdot 3^{-9} \cdot 3^6$
7) $\frac{1}{n^7} \cdot \frac{1}{n^{-4}}$
8) $\frac{1}{x^{-4}} \cdot \frac{1}{x^{-2}}$

Unit 8 Section 2

Scientific Notation (pg. 159 – 162)

Vocabulary:

<u>Scientific Notation</u> – A number expressed in the form a x 10^n, where n is an integer and $1 < a < 10$.

Practice (Writing a number in scientific notation):

Steps:

1) Moving the decimal for a whole number – Place a decimal at the end of the number and count how many spaces it would take to get right behind the first number. (Move to the left)

   Ex) 902,700,000,000,000

   ↑ ↑

   Where decimal ends   Where decimal starts

   (14 spaces to the left)

   Moving the decimal for a number that already has a decimal - Count how many spaces it would take to get right behind the first non-zero number. (Move to the right)

   Ex) 0.00000000010581

   ↑ ↑

   Where decimal starts   Where decimal ends

   (10 spaces to the right)

2) Cross out any zeros in the number, unless the zero is in between non-zero numbers. Then you are not allowed to cross out the zero(s).

   Ex) 902,700,000,000,000 = 9.027

   Ex) 1.0581

3) Set the number to the product of 10.

   Ex) 9.027 x 10

   Ex) 1.0581 x 10

4) The exponent for 10 will depend on the direction you went with your decimal, as well as how many spaces. If you moved your decimal to the left, your exponent will be positive. If you moved your decimal to the right, your exponent will be negative.

   Ex) 9.027×10^{14}

   Ex) 1.0581×10^{-10}

Practice (Writing a number in standard notation):

Just do the opposite! If the exponent is positive, it was moved the left to write it in scientific notation. So do the opposite. Go to the right to make it the original form. If the exponent is negative, it was moved to the right to write it in scientific notation. So do the opposite. Go to the left to make it the original form.

Ex) $9.12 \times 10^{8} = 912{,}000{,}000$

Ex) $4.0953 \times 10^{-10} = 0.00000000040953$

Extra Practice:

For questions 1 – 4, write the numbers in scientific notation.

1) 1,250,004,500,000,000,000
2) –284,500,000,100,000,000
3) 0.00000000000024
4) –0.00050012

For questions 5 – 8, write the numbers in standard notation.

5) 3.73×10^{-6}
6) 4.21×10^{9}
7) -5.72×10^{-5}
8) -1.494×10^{8}

For questions 9 & 10, place in order from least to greatest.

9) $10^{-3}, 5 \times 10^{-3}, 8 \times 10^{-2}, 4 \times 10^{-1}$
10) $5.01 \times 10^{-4}, 4.8 \times 10^{-3}, 5.2 \times 10^{-2}, 5.6 \times 10^{-2}$

For questions 11 & 12, simplify and write your answer in scientific notation.

11) $2\left(6.1 \times 10^{-6}\right)$
12) $7\left(9 \times 10^{6}\right)$

Unit 8 Section 8

Exponential Growth and Decay

Vocabulary:

Exponential Growth – A situation modeled with a function of the form y = ab^x, where a > 0 and b > 1.

Y = ab^x
A = starting amount (when x = 0)
B = the base, which is greater than 1(called the growing factor)
X = the exponent

Growth Factor – The number b in an exponential function, where b > 1.

Ex) y = $25\ 3^x$

Compound Interest - Interest paid on both the principal and the interest that has already been paid.

Interest Period – The length of time that interest is calculated.

Exponential Decay – A situation modeled with a function of the form y = ab^x, where a > 0 and 0 < b < 1.

Decay Factor – The number b in an exponential function, where 0 < b < 1.

Ex) y = $4\ 0.5^x$

Practice (Finding the initial amount and the growth/decay factor of each function):

Ex) f(x) = 30 • 3.128^x
Initial amount = 30
Growth amount = 3.128

Ex) g(x) = 0.125^x
Initial amount = 1 (since no value is written for the a value, we place a 1 to not change the values of the problem)
Decay amount = 0.125

Practice (Modeling exponential growth and decay):

Ex) A town with a population of 5,000 grows 5% per year. Find the population at the end of 10 years. What if the population decreased 3% each year? Use the table below to fine the amounts. Round to the nearest whole number.

A(initial amount)	B(growth/decay factor)	X(number of increases)	Y(new amount)
5,000	100% + 5% = 105% = 1.05	10	5,000 • 1.05^{10} = 8,144
5,000	100% - 3% = 97% = .97	10	5,000 • $.97^{10}$ = 3,687

Practice (Finding compound interest):

Ex) A total of $12,000 is deposited in an account paying 9% interest compounded annually (once a year). Find the account balance after 5 years. Round to the nearest cent.

Y = ab^x
Y = the balance
A = the initial deposit ($12,000)
B = 100% + 9% = 109% = 1.09
X = number of interest periods (5)

Y = 12,000 • 1.09^5
Y = $18,463.49

Extra Practice:

For questions 1 – 4, find the initial amount and the growth/decay factor of each function.

1) g(x) = 15 • 10^x
2) f(x) = 0.45^x
3) y = 1,000,000 • 1.89^x
4) h(x) = $15,000 • 0.2^x

For questions 5 – 8, create a chart to find your new amount (y) for exponential growth and decay. Round your answers to the nearest whole number.

5 & 6) A town with a population of 8,000 grows 3% per year. Find the population at the end of 15 years. What if the population decreases 1% each year?

7 & 8) A city has a population of 1,245,600, which grows 2% per year. Find the population at the end of 10 years. What if the population decreased at 3% each year?

For questions 9 & 10, find the compound interest. Round your answer to the nearest cent.

9) You deposit $25,000 in a trust fund that pays 8.75% interest compounded annually (once a year), on the day that your grandchild was born. What will the account balance of this account be on your grandchild's 25th birthday?

10) $250 is deposited in an account that pays 5.5% interest compounded yearly. What is the balance after 8 years?

Exponential Expression	Factors	Values
2^6	2 • 2 • 2 • 2 • 2 • 2	64
2^5	2 • 2 • 2 • 2 • 2	32
2^4	2 • 2 • 2 • 2	16
2^3	2 • 2 • 2	8
2^2	2 • 2	4
2^1	2	2
2^0	1	1

As the exponential expression decreases by *1*, the value is cut in ½ each time. So lets continue the chart, decreasing the exponential expression by *1* to see what happens the values.

Exponential Expression	Factors	Values	Decimal Values
2^{-1}	$\frac{1}{2^1}$	$\frac{1}{2}$	0.5
2^{-2}	$\frac{1}{2^2}$	$\frac{1}{4}$	0.25
2^{-3}	$\frac{1}{2^3}$	$\frac{1}{8}$	0.125

Practice (Zero as an Exponent):

Ex) $3^0 = 1$
Ex) $-8^0 = 1$
Ex) $(-2.25)^0 = 1$
Ex) $\left(\frac{3}{4}\right)^0 = 1$
Ex) $-(5.6)^0 = -1$
Ex) $4xy^0 = 4x$

Practice (Negative Exponents):

Ex) $3^{-5} = \frac{1}{3^5}$

Ex) $(-6)^{-7} = \frac{1}{(-6)^7}$

Ex) $(-9)^{-1} = \frac{1}{(-9)^1}$

Ex) $x^{-4} = 1\frac{x^{-4}}{1} = \frac{1}{x^4}$

Ex) $2x^{-1} = 2\frac{x^{-1}}{1} = \frac{2}{x^1} = \frac{2}{x}$

Ex) $3xy^{-5} = 3x\cdot\frac{1}{y^5} = 3\frac{x}{1}\cdot\frac{1}{y^5} = 3\frac{x}{y^5}$

Ex) $\frac{1}{x^{-4}} = 1\cdot x^4 = x^4$

For questions 1 – 8, simplify each expression.

1) $2x^0$
2) $(-53)^0$
3) $\frac{1}{2^0}$
4) -4^{-5}
5) $(-2)^{-6}$
6) $\frac{1}{d^{-1}}$
7) $x^5 y^0$
8) $3\frac{x^5}{y}$

For questions 9 & 10, evaluate the expression for r = -3 and s = 5.

9) $\frac{1}{r^{-4} s^5}$
10) $2^{-6} r^4 s^{-2}$