Unit 9--Polynomials and Factoring

Section 1 – Adding and Subtracting Polynomials (pg. 163 – 170)

Vocabulary:

Monomial – An expression that is a number, a variable, or a product of a number and one or more variables.

Ex) 8 , x, and $@2xy^2$**,** $\frac{m}{5}$ **are monomials.**

Be careful! $\frac{5}{m}$ **is NOT a monomial.**

Degree of a Monomial – The sum of the exponents of the variables of a monomial. For a nonzero constant, the degree is zero. Zero does not have a degree.

Ex) $4x^3 + y^2$ **= The exponents are 3 and 2, their sum is 5. So the degree is 5.**
Ex) -8 = The degree of a nonzero constant is 0.
Ex) 3x = The exponent is 1. So the degree is 1.

Polynomial – The sum or difference of two or more monomials. A quotient with a variable in the denominator is not a polynomial.

Ex) $4x^2$**, 5x + 8,** $x^2 + 2x @35$ **are polynomials.**

Standard Form of a Polynomial – The form of a polynomial in which the degree of the terms decreases from left to right (descending order).

Ex) $x^3 @4x^2 + 3$ **is in standard form because the degrees decrease from left to right.**

Degree of a Polynomial – The degree of the term with the greatest exponent for a polynomial in one variable.

Ex) $2x^4 + 5x^3 @x^2 + 3x @8$ **had a degree of 4, because it belongs to the monomial with the greatest exponent.**

Binomial - A polynomial of two terms.

Ex) 4x + 3 is a binomial.

Trinomial – A polynomial of three terms.

Ex) $x^2 @x @42$

Classifying Polynomials

Polynomial	Degree	Classified by Degree	Classified by Term
4	0	Constant	Monomial
-5x	1	Linear	Monomial
6x + 2	1	Linear	Binomial
$x^2 @6x + 9$	2	Quadratic	Trinomial
$9x^3 @2x$	3	Cubic	Binomial
$2x^4 @7x^3 @5x + 1$	4	Quartic	Polynomial

Unit 9 Section 7
Factoring Special Cases

Vocabulary:

Perfect Square Trinomial – You can factor a perfect square trinomial into identical binomial factors. To recognize a perfect square trinomial, the first and last terms must be positive and perfect squares, $a^2 \text{ and } b^2$, and the middle term must be twice the product of *a* and *b*.

Rule:

$a^2 + 2ab + b^2 = (a + b)^2$

$a^2 - 2ab + b^2 = (a - b)^2$

You can factor perfect square trinomials like in previous lessons, or you can find the perfect square trinomial and factor it faster.

Ex) $4x^2 + 12x + 9$
$2x \cdot 2x \qquad 3 \cdot 3$
$2 \cdot 2x \cdot 3 = 12x$
This is a perfect square trinomial because twice the product of a and b equaled the middle term.

So the factored form would be: $(2x + 3)^2$

Check: $(2x + 3)(2x + 3) = 4x^2 + 6x + 6x + 9$
$= 4x^2 + 12x + 9$

Ex) $4x^2 + 20x + 9$
$2x \cdot 2x \qquad 3 \cdot 3$
$2 \cdot 2x \cdot 3 = 20x$
This is not a perfect square trinomial because twice the product of a and b equals 12x, not 20x.

Ex) $x^2 - 4x + 4$
$x \cdot x \quad -2 \cdot -2$
$2 \cdot x \cdot -2 = -4x$

So the factored form would be: $(x - 2)^2$

Check: $(x - 2)(x - 2) = x^2 - 2x - 2x + 4$
$= x^2 - 4x + 4$

Ex) $25y^2 - 10y + 1$
$5y \cdot 5y \quad -1 \cdot -1$
$2 \cdot 5y \cdot -1 = -10y$

So the factored form would be: $(5y - 1)^2$

Check: $(5y - 1)(5y - 1) = 25y^2 - 5y - 5y + 1$
$= 25y^2 - 10y + 1$

Rule:

The difference of two squares: $a^2 - b^2 = (a + b)(a - b)$

Practice (Factoring the difference of two squares):

Ex) $4x^2 - 4 = (2x)^2 - 2^2$

$= (2x + 2)(2x - 2)$

Check: $(2x + 2)(2x - 2) = 4x^2 - 4x + 4x - 4$

$= 4x^2 - 4$

Ex) $y^2 - 25 = y^2 - 5^2$

$= (y + 5)(y - 5)$

Check: $(y + 5)(y - 5) = y^2 - 5y + 5y - 25$

$= y^2 - 25$

Practice (Factoring out a common difference):

Ex) $5y^2 - 45 = 5(y^2 - 9)$

$= 5(y + 3)(y - 3)$

Check: $5(y + 3)(y - 3) = 5(y^2 - 3y + 3y - 9)$

$= 5(y^2 - 9)$

$= 5y^2 - 45$

Extra Practice:

For questions 1 - 3 , factor the perfect square trinomial.

1) $4y^2 + 4t + 1$

2) $9x^2 - 12x + 4$

3) $16x^2 + 8x + 1$

For questions 4 – 6, factor the difference of the two squares.

4) $x^2 - 36$

5) $9z^2 - 100$

6) $4y^2 - 144$

For questions 7 – 9, factor out a common factor.

7) $3c^2 - 9$

8) $4x^2 - 16$

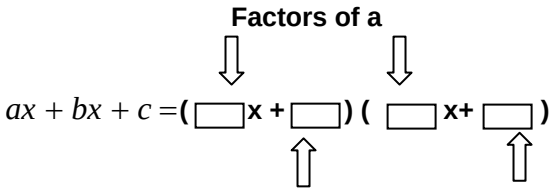
9) $5y^2 - 245$

Unit 9 Section 6

Factoring Trinomials of the Type $ax^2 + bx + c$

Rule:

To factor a trinomial of the form $ax^2 + bx + c$, use the following pattern.



The goal is to find a combination of factors of *a* and *c* so that the outer and inner products add up to the middle term *bx*.

Practice (Factoring trinomials of the type $ax^2 + bx + c$:

Ex) $6x^2 + 17x + 5$

Factors for $6x^2$ (a): 1 O6, 2 O3

Factors for 5 (c): 1 O5

Sum of factors to be 17x (bx)

After some experimentation, you can find that the factorization is: $(2x + 5)(3x + 1)$

Check: $(2x + 5)(3x + 1) = 6x^2 + 2x + 15x + 5$

$= 6x^2 + 17x + 5$

Ex) $x^2 - 7x + 12$

Factors for x^2 (a):

Factors for 12 (c): 1 O12, 2 O6, 3 O4

Sum of factors to be -7x (bx)

After some experimentation, you can find that the factorization is: $(x - 3)(x - 4)$

Check: $(x - 3)(x - 4) = x^2 - 4x - 3x + 12$

$= x^2 - 7x + 12$

Ex) $2x^2 + x - 15$

Factors of $2x^2$ (a): 1 O2

Factors for -15 (c): 1 O-15, -1 O15, 3 O-5, -3 O5

Sum of factors to be 1x (bx)

After some experimentation, you can find that the factorization is: $(2x - 5)(x + 3)$

Check: $2x - 5)(x + 3) = 2x^2 + 6x - 5x - 15$

$= 2x^2 + 1x - 15$

$= 2x^2 + x - 15$

Ex) $x^2 - 2x - 8$

Factors of x^2 (a):	Factors for -8 (c):	Sum of factors to be -2x (bx)
	$1 \cdot -8$	
	$-1 \cdot 8$	
	$2 \cdot -4$	
	$-2 \cdot 4$	

After some experimentation, you can find that the factorization is: $(x + 2)(x - 4)$

Check: $(x + 2)(x - 4) = x^2 - 4x + 2x - 8$

$= x^2 - 2x - 8$

Extra Practice:

For questions 1 – 8, factor the trinomials of the type $ax^2 + bx + c$.

1) $x^2 + 10x + 25$

2) $4t^2 + 4t + 1$

3) $s^2 - 5s + 6$

4) $9x^2 - 12x + 4$

5) $x^2 + x - 20$

6) $5u^2 + 13u - 6$

7) $t^2 - t - 6$

8) $9z^2 - 3z - 2$

Unit 9 Section 5

Factoring Trinomials of the Type $x^2 + bx + c$

Rule:

To factor a trinomial of the form $x^2 + bx + c$, you must have two numbers that have a sum of b and a product of c.

Practice (Factoring $x^2 + bx + c$):

Ex) $x^2 + 12x + 36$

Find the factors of 36. Identify the pair that has the sum of 12.

Factors of 36	Sum of Factors
1 and 36	37
2 and 18	20
3 and 12	15
4 and 9	13
6 and 6	12

$x^2 + 12x + 36 = (x + 6)(x + 6)$

Check:

$(x + 6)(x + 6) = x^2 + 6x + 6x + 36$

$= x^2 + 12x + 36$

Ex) $a^2 + 16a + 28$

Find the factors of 28. Identify the pair that has the sum of 16.

Factors of 28	Sum of Factors
1 and 28	29
2 and 14	16
4 and 7	11

$a^2 + 16z + 28 = (a + 2)(a + 14)$

Check:

$(a + 2)(a + 14) = a^2 + 14a + 2a + 2\ 14$

$= a^2 + 16a + 28$

Rule:

To factor a trinomial of the form $x^2 @ bx + c$, you must look at the middle term. If the middle term is negative, you need to inspect the negative factors of *c* to find the factors of the trinomial.

Practice Factoring $x^2 @ bc + c$:

Ex) $x^2 @ 14x + 49$

Since the middle term is negative, find the negative factors of 49. Identify the pair that has the sum of -14.

Factors of 49	Sum of Factors
-1 and -49	-50
-7 and -7	-14

$x^2 @14x + 49 = x @7^{a} x @7^{a}$

Check:

$x @7^{a} x @7^{a} = x^2 @7x @7x + 49$

$= x^2 @14x + 49$

Rule:

If there is a negative constant, you need to look at pairs of negative and positive factors of c.

Practice (Factoring with Negative C):

Ex) $x^2 + 4x @32$

Identify the pair of factors of -32 that has the sum of 4.

Factors of -32	Sum of Factors
1 and -32	-31
32 and -1	31
2 and -16	-14
16 and -2	14
4 and -8	-4
8 and -4	4

$x^2 + 4x @32 = x + 8^{a} x @4^{a}$

Check:

$x + 8^{a} x @4^{a} = x^2 @4x + 8x @32$

$= x^2 + 4x @32$

Ex) $x^2 @6x @16$

Identify the pair of factors of -16 that has the sum of -6.

Factors of -16	Sum of Factors
1 and -16	-15
16 and -1	15
2 and -8	-6
8 and -2	6

$x^2 @6x @16 = x + 2^{a} x @8^{a}$

Check:

$x + 2^{a} x @8^{a} = x^2 @8x + 2x @16$

$= x^2 @6x @16$

Rule:

When factoring trinomials with two variables the first term is the square of the first variable, the middle term includes both variables and the last term includes the square of the second variable.

Practice (Factoring with Two Variables):

Ex) $x^2 + 12xy + 35y^2$

Identify the pair of factors of 35 that has the sum of 12.

Factors of 35	Sum of Factors
1 and 35	36
5 and 7	12

$x^2 + 12xy + 35y^2 = (x + 5y)(x + 7y)$

Check:

$(x + 5y)(x + 7y) = x^2 + 7xy + 5xy + 35y^2$

$= x^2 + 12xy + 35y^2$

Extra Practice:

For questions 1 and 2, factor the trinomial in the form $x^2 + bx + c$.

1) $x^2 + 15x + 50$

2) $x^2 + 12x + 20$

For questions 3 and 4, factor the trinomial in the form $x^2 \pm bx + c$.

3) $x^2 \pm 16x + 39$

4) $x^2 \pm 18x + 72$

For questions 5 and 6, factor the trinomial with a negative *c*.

5) $x^2 + 6x \pm 40$

6) $b^2 \pm 4b \pm 45$

For questions 7 and 8, factor the trinomial with two variables.

7) $x^2 + 15xy + 44y^2$

8) $a^2 \pm 10ab \pm 24b^2$

Unit 9 Section 4
Multiplying Special Cases

Rule: The Square of a Binomial

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

Practice (Squaring a Binomial):

Ex) $(x+3)^2 = x^2 + 2(x)(3) + 3^2$

$= x^2 + 6x + 9$

Ex) $(x-4)^2 = x^2 - 2(x)(4) + 4^2$

$= x^2 - 8x + 16$

Ex) $(2x+5)^2 = (2x)^2 + 2(2x)(5) + 5^2$

$= 4x^2 + 20x + 25$

Ex) $(3x-2)^2 = (3x)^2 - 2(3x)(2) + 2^2$

$= 9x^2 - 12x + 4$

Rule: The Difference of Squares

$$(a+b)(a-b) = a^2 - ab + ab - b^2$$

$$= a^2 - b^2$$

Practice (Finding the Difference of Squares):

Ex) $(r+6)(r-6) = r^2 - 6^2$

$= r^2 - 36$

Ex) $(x^3+4)(x^3-4) = (x^3)^2 - 4^2$

$= x^6 - 16$ Remember, when raising a power by a power you multiply

Ex) $(6x^2+y^5)(6x^2-y^5) = (6x^2)^2 - (y^5)^2$

$= 36x^4 - y^{10}$

Extra Practice:

For questions 1 – 5, square the binomials.

1) $\left(q+4\right)^2$

2) $\left(r-3\right)^2$

3) $\left(2x+2\right)^2$

4) $\left(5r-6\right)^2$

5) $\left(7t-6p\right)^2$

For questions 6 – 10, find the difference of the squares.

6) $\left(q+12\right)\left(q-12\right)$

7) $\left(x^2+9\right)\left(x^2-9\right)$

8) $\left(7z^3+2\right)\left(7z^3-2\right)$

9) $\left(3x^4+y^5\right)\left(3x^4-y^5\right)$

10) $\left(6w^3+4v^4\right)\left(6w^3-4v^4\right)$

Unit 9 Section 3
Multiplying Binomials (pg. 173 – 176)

Any time you are trying to multiply binomials together, the *FOIL* Method is a way to help organize your information.

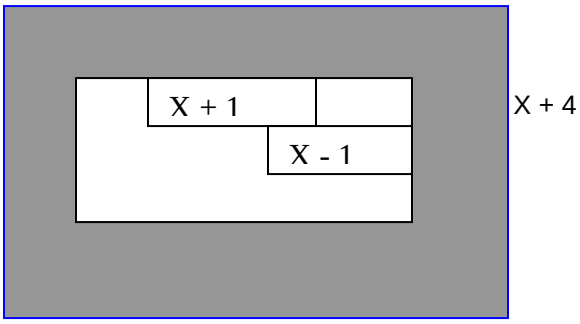
F - first
O - outer
I - inner
L – last

Ex) $x @5^{a}\ x + 10^{a}$

F = x • x = x^2
O = x • 10 = 10x
I = -5 • x = -5x
L = -5 • 10 = -50

$x^2 + 10x @5x @50$ (Wrote out all of our values)
$x^2 @5x @50$ (Combine like terms)

Ex) Find the area of the shaded region only. Write your answer as a polynomial in standard form.



Area of big rectangle:
A = b • h
A = $x + 7^{a}\ x + 4^{a}$

F = X • X = X^2
0 = X • 4 = 4X
I = 7 • X = 7X
L = 7 • 4 = 28

$x^2 + 4x + 7x + 28$
$x^2 + 11x + 28$

Area of small rectangle:
A = b • h
A = $x + 1^{a}\ x @1^{a}$

F = x • x = x^2
O = x • -1 = -1x
I = 1 • x = 1x
L = 1 • -1 = -1

$x^2 @1x + 1x @1$
$x^2 @1$

$x^2 + 11x + 28 @^{b}\ x^2 @1^{c}$ Subtract the areas to take out the white section.

$x^2 + 11x + 28 @x^2 + 1$ Use the distributive property.
$11x + 29$ Combined like terms.

Extra Practice:

For questions 1 – 5, simplify using the *FOIL* Method.

1) $3x \cdot 5^{\square} 2x + 1^{\square}$

2) $x + 6^{\square 2}$

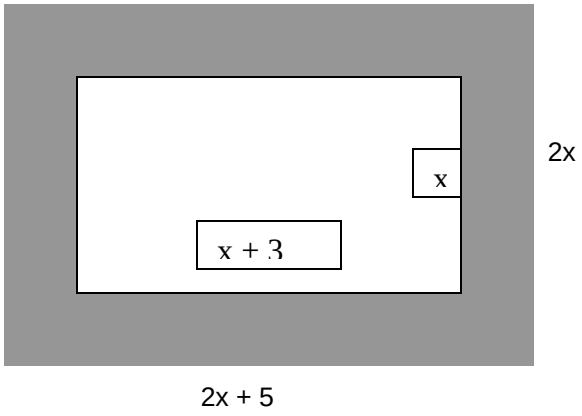
3) $2x + 3^{\square} 2x \cdot 3^{\square}$

4) $4x^{3} \cdot 3^{2}$

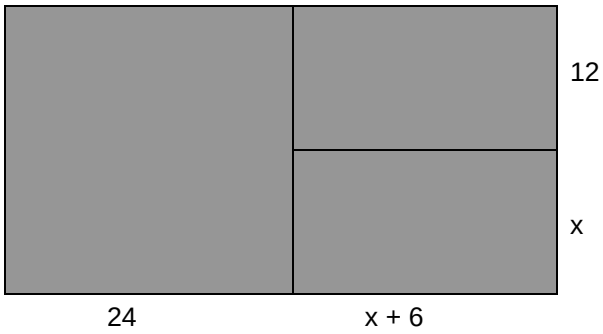
5) $3x + 4^{\square} 2x \cdot 1^{\square}$

For questions 6 – 8, find the area of the shaded region only. Write your answer as a polynomial in standard form.

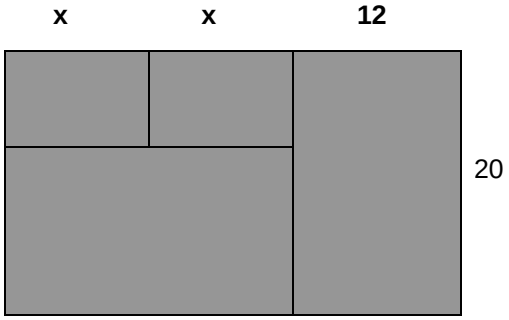
6)



7)



8)



Unit 9 Section 2

Multiplying and Factoring (pg. 170 – 173)

When multiplying a polynomial to a monomial, the Distributive Property is used to help organize the information.

Hint: Take the monomial (connected to the parentheses by multiplication) and multiply to each part of the polynomial.

Remember, when you multiply with the same base, you add the exponents. If there is no exponent written (there is just a variable) the exponent is really 1.

Practice (Using the Distributive Property):

Ex) $4(x+5)$

$4 \cdot x = 4x$
$4 \cdot 5 = 20$
$\Rightarrow 4x + 20$

Ex) $8(3x^2 + 2x + 1)$

$8 \cdot 3x^2 = 24x^2$
$8 \cdot 2x = 16x$
$8 \cdot 1 = 8$
$\Rightarrow 24x^2 + 16x + 8$

Ex) $3x(2x - 3)$

$3x \cdot 2x = 6x^2$
$3x \cdot -3 = -9x$
$\Rightarrow 6x^2 - 9x$

Ex) $-2y^2(3y^2 + 4y - 1)$

$-2y^2 \cdot 3y^2 = -6y^4$
$-2y^2 \cdot 4y = -8y^3$
$-2y^2 \cdot -1 = 2y^2$
$\Rightarrow -6y^4 - 8y^3 + 2y^2$

Ex) $2(t^2 + 5) - 3(t^2 + 5) + 5(t^2 + 5)$

$2 \cdot t^2 = 2t^2$ $\quad -3 \cdot t^2 = -3t^2$ $\quad 5 \cdot t^2 = 5t^2$
$2 \cdot 5 = 10$ $\quad -3 \cdot 5 = -15$ $\quad 5 \cdot 5 = 25$

$2t^2 + 10 - 3t^2 - 15 + 5t^2 + 25$ Combined all the answers from using the distributive property.
$4t^2 + 20 + 2t$ Combined like terms.
$4t^2 + 2t + 20$ Put in standard form.

Practice (Factoring Out Monomials):

Ex) $4x^3 + 12x^2 + 24x$

$4x^3 = 4 \cdot x \cdot x \cdot x$
$12x^2 = 4 \cdot 3 \cdot x \cdot x$
$24x = 4 \cdot 6 \cdot x$
Greatest Common Factor: 4x
Re-Write the equation taking 4x out to be in the front.
$4x(x^2 + 3x + 6)$

Ex) $3x^4 @ 12x^3 + 21x^2$

$3x^4 = 3 \cdot x \cdot x \cdot x \cdot x$

$@12x^3 = 3 \cdot @4 \cdot x \cdot x \cdot x$

$21x^2 = 3 \cdot 7 \cdot x \cdot x$

Greatest Common Factor: $3x^2$

Re-Write the equation taking $3x^2$ out to be in the front.

$3x^2 \left(1x^2 @ 4x + 7 \right)$

$3x^2 \left(x^2 @ 4x + 7 \right)$

Extra Practice:

For questions 1 – 5, use the Distributive Property to solve. Simplify your answer and place in standard form.

1) $3x^2 \left(9x @ 1 \right)$

2) $@10y \left(y^2 @ 10y \right)$

3) $2x \left(@3x^2 + 12x @ 15 \right)$

4) $@10u \left(u + 1 \right) + 8u \left(u @ 1 \right) @ 3u \left(u + 6 \right)$

5) $@ \left(5x^2 @ 1 \right) @ \left(@3x^2 + 5 \right) @ \left(x^2 @ x \right)$

For questions 6 – 10, find the Greatest Common Factor and re-write each equation.

6) $14x^4 + 7x^3 + 28x$

7) $36x^2 y^2 + 16x^2 y + 84xy^2$

8) $@a^3 @ 4a$

9) $@3x^5 + 15x^3 + 6x^2$

10) $20x^5 @ 8x^4 + 4x^3$

Unit 9 Section 8
Factoring by Grouping

Sometimes polynomials with more than three terms can be factored by grouping. You can use the Distributive Property if two groups of terms have the same factor.

Practice (Factoring by grouping):

Ex) $x^3 @2x^2 @3x + 6 = \overset{\smile}{x^3 @2x^2}\overset{\frown}{} @ \overset{a}{3x @6}$ Group the terms

$= x^2\ x @2^{a} @3\ x @2^{a}$ Factor groups

$= \overset{a\smile}{x @2}\ \overset{\frown}{x^2 @3}$ $x @2$ from the groups and $\overset{\smile}{x^2 @3}\overset{\frown}{}$ from the outside factors.

Check: $x @2^{a\smile}\ x^2 @3\overset{\frown}{} = x^3 @3x @2x^2 + 6$

$= x^3 @2x^2 @3x + 6$

Ex) $x^3 @x^2 + 2x @2 = \overset{\smile}{x^3 @x^2}\overset{\frown}{} + \overset{a}{2x @2}$ Group the terms

$= x^2\ x @1^{a} + 2\ x @1^{a}$ Factor groups

$= \overset{a\smile}{x @1}\ \overset{\frown}{x^2 + 2}$ $x @1$ from the groups and $\overset{\smile}{x^2 + 2}\overset{\frown}{}$ from the outside factors.

Check: $x @1^{a\smile}\ x^2 + 2\overset{\frown}{} = x^3 + 2x @1x^2 @2$

$= x^3 @1x^2 + 2x @2$

$= x^3 @x^2 + 2x @2$

Extra Practice:

For questions 1 – 5, factor by grouping.

1) $2x^3 @x^2 @6x + 3$

2) $6 + 2x @3x^3 @x^4$

3) $x^3 + 5x^2 @5x @25$

4) $5x^3 @10x^2 + 3x @6$

5) $x^5 + 2x^3 + x^2 + 2$

Practice (Writing polynomials in standard form):

Hint: Make sure to keep the sign in front of each monomial!

Ex) $4x^2 @ 5x^3 @ 2 + 3x$ [$@5x^3 + 4x^2 + 3x @ 2$

Practice (Adding polynomials vertically):

Hint: Match up like terms and add them together!

Ex) $5x^3 @ 7x^2 @ 3$

\+ $x^3 + 2x^2 @ x + 8$

$6x^3 @ 5x @ x + 5$ (Because the –x did not have another term to match with, you just bring it down into the answer)

Practice (Adding polynomials horizontally):

Hint: Match up like terms and add them together

Ex) $@(x^3 @ 2) + (4x^3 @ 2x)$

$@x^3 + 2 + 4x^3 @ 2x$ (Get rid of the parentheses by using the distributive property, remember that a double negative makes a positive)

$3x^3 + 2 @ 2x$ (Combine like terms. Remember that $@x^3 = @1x^3$)

$3x^3 @ 2x + 2$ (Put answers in standard form)

Practice (Subtracting polynomials vertically):

Hint: Put the second polynomial in parentheses and use the distributive property!

Ex) $2x^2 + 3x @ 4$

$@(x^2 + x @ 1)$ [$2x^2 + 3x @ 4$

$@x^2 @ x + 1$ (Used the distributive property on second polynomial)

$1x^2 + 2x @ 3$

$x^2 + 2x @ 3$ (Do not need to write the one in front of the x^2)

Practice (Subtracting polynomials horizontally):

Hint: Use the distributive property to eliminate parentheses!

Ex) $@(5x^2 @ 1) @ (@3x^2 + 5) @ (x^2 @ x)$

$@5x^2 + 1 + 3x^2 @ 5 @ x^2 + x$ (Used the distributive property)

$@3x^2 @ 4 + x$

$@3x^2 + x @ 4$ (Put in standard form)

Extra Practice:

For questions 1 & 2, find the degree of the monomial.

1) $9x^3 y^2 z^4$

2) 14

For questions 3 & 4, classify the polynomial by degree and by number of terms.

3) $3x^2 + 6x + 1$

4) -5x – 4

For questions 5 – 7, add the polynomials.

5) $@4x^3 @2x^2 + x @5$
$\underline{+\ 2x^3 + 3x + 4}$

6) $(5x^4 @2x + 7) + (@3x^4 + 6x^2 @5)$

7) $(12x^3 + 2x^2 @4) + (9x^2 + 3x @8)$

For questions 8 – 10, subtract the polynomials.

8) $7x^4 @x^2 + 2$
$\underline{@\ (3x^4 @4x^2 + 3x)}$

9) $(15x^2 @6) @ (@8x^3 @14x^2 @17)$

10) $(15x^4 @18x @19) @ (13x^4 @5x + 15)$

For questions 11 & 12, use addition and subtraction to solve the polynomials.

11) $@(x^3 @2) + (4x^3 @2x) @ (2x^2 + 3)$

12) $(y^3 + 1) @ (y^2 + 1) + (3y @7)$